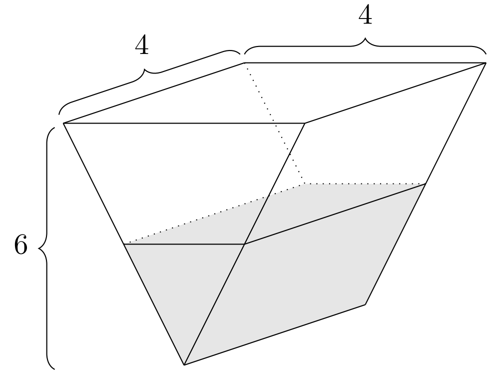
4. [10 points]

a. [5 points]
Sam is pouring concrete into a hole in the shape of a triangular prism. The hole is 4 meters wide, 4 meters long, and 6 meters deep at its deepest point. A partially filled hole with the correct dimensions is shown to the right.



Sam is looking down into the hole and observes that the rectangular top surface of the concrete is growing at a rate of 0.8 meters squared per minute. Find the rate at which the depth of the concrete is growing. *Include units.*

Solution: Let w denote the width of the surface rectangle and h denote the depth of the concrete. By similar triangles, $w = \frac{2}{3}h$. Then the area, $A(h)$, is

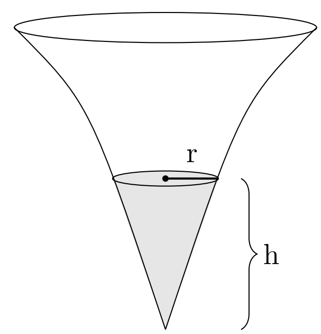
$$A(h) = 4\left(\frac{2}{3}h\right) = \frac{8}{3}h.$$

Taking the derivative with respect to time we have $\frac{dA}{dt} = \frac{8}{3}\frac{dh}{dt}$, so $\frac{dh}{dt} = 0.8\left(\frac{3}{8}\right) = 0.3$.

Answer: 0.3 **meters per minute**

b. [5 points]
Donna is pouring concrete into a different hole, which is in the shape of a horn as shown to the right. When the concrete has been poured to a depth of h meters and its surface has radius r, the volume of the poured concrete is given by

$$V = \frac{\pi}{7}r^2h.$$



When the depth of the concrete that has been poured is 0.8 meters, the radius of its surface is 0.5 meters, the radius is growing at a rate of 5 meters per hour, and the volume is growing at a rate of 2 cubic meters per hour. How fast is the depth changing? *Include units.*

Solution:

$$\frac{dV}{dt} = \frac{\pi}{7}\left(2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right)$$
$$2 = \frac{\pi}{7}\left(2(0.5)(0.8)(5) + (.5)^2\frac{dh}{dt}\right)$$
$$\frac{14}{\pi} = 4 + 0.25\frac{dh}{dt}$$
$$\frac{14}{\pi} - 4 = 0.25\frac{dh}{dt}$$
$$4\left(\frac{14}{\pi} - 4\right) = \frac{dh}{dt}$$

Answer: $4\left(\frac{14}{\pi} - 4\right) \approx 1.825$ **meters per hour**