6. [13 points] The following are tables of values for two differentiable functions \( f(x) \) and \( g(x) \) and their derivatives. Missing values are denoted by a “?”. Assume that each of these functions is defined for all real numbers, that \( f'(x) \) and \( g'(x) \) are continuous, and that \( g(x) \) is invertible.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>2</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>( x )</th>
<th>-1</th>
<th>1</th>
<th>3</th>
<th>7</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>-1</td>
<td>?</td>
<td>0</td>
<td>-2</td>
<td>?</td>
<td>( g(x) )</td>
<td>-4</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>( f'(x) )</td>
<td>1</td>
<td>4</td>
<td>-1</td>
<td>?</td>
<td>1</td>
<td>( g'(x) )</td>
<td>7</td>
<td>?</td>
<td>3</td>
<td>4</td>
<td>?</td>
</tr>
</tbody>
</table>

a. [4 points] For each of the following, find the value exactly. If there is not enough information to find the quantity, write NEI.

i. [2 points] Let \( z(x) = f(g(x)) \). Find \( z'(3) \).

**Answer:** \( z'(3) = 12 \)

ii. [2 points] Let \( j(x) = g^{-1}(x) \). Find \( j'(7) \).

**Answer:** \( j'(7) = \text{NEI} \)

b. [2 points] Use a left-hand Riemann sum with three equal subintervals to estimate \( \int_{-1}^{11} g(x) \, dx \). Write out all the terms in your sum.

**Answer:** \( 4(-4 + 2 + 6) \)

c. [1 point] Is your answer in part b. an overestimate or an underestimate? Circle your answer. If there is not enough information circle NEI.

**Answer:** OVERESTIMATE

UNDERESTIMATE

NEI

d. [4 points] The function \( f(x) \) has two critical points, at \( x = 2.5 \) and \( x = \pi \). These are the only critical points of \( f(x) \). For each critical point, decide if it is a local max, local min, neither, or if there is not enough information to determine this (NEI). Circle your answers.

**Answer:** \( x = 2.5 \) is a: LOCAL MIN

LOCAL MAX

NEITHER

NEI

**Answer:** \( x = \pi \) is a: LOCAL MIN

LOCAL MAX

NEITHER

NEI

e. [2 points] On which of the following interval(s) \( f(x) \) have an inflection point? Circle all correct answers.

\[ [0, 3] \quad [2, 3] \quad [3, 9] \]

**Solution:** We know that for \( f(x) \) to have an inflection point \( p \), the sign of \( f''(x) \) must change at \( p \). The sign of \( f''(x) \) must change during the interval \([0, 3]\), but it does not happen at a single point – for example, this can occur if \( f''(x) \) is first positive, and then zero over an interval, and then negative. So the correct answer is that none of the intervals must have an inflection point; however full credit was also given if only \([0, 3]\) was circled.