6. [13 points] The following are tables of values for two differentiable functions $f(x)$ and $g(x)$ and their derivatives. Missing values are denoted by a "?". Assume that each of these functions is defined for all real numbers, that $f'(x)$ and $g'(x)$ are continuous, and that $g(x)$ is invertible.

x	0	2	3	6	9
$f(x)$	-1	?	0	-2	?
$f'(x)$	1	4	-1	?	1

x	-1	1	3	7	11
$g(x)$	-4	1	2	6	7
$g'(x)$	7	?	3	4	?

a. [4 points] For each of the following, find the value exactly. If there is not enough information to find the quantity, write NEI.

i. [2 points] Let $z(x) = f(g(x))$. Find $z'(3)$.

Answer: $z'(3) =$ **12**

ii. [2 points] Let $j(x) = g^{-1}(x)$. Find $j'(7)$.

Answer: $j'(7) =$ **NEI**

b. [2 points] Use a left-hand Riemann sum with three equal subintervals to estimate $\int_{-1}^{11} g(x)\,dx$. Write out all the terms in your sum.

Answer: $4(-4+2+6)$

c. [1 point] Is your answer in part **b.** an overestimate or an underestimate? Circle your answer. If there is not enough information circle NEI.

Answer: OVERESTIMATE $\boxed{\text{UNDERESTIMATE}}$ NEI

d. [4 points] The function $f(x)$ has two critical points, at $x = 2.5$ and $x = \pi$. These are the only critical points of $f(x)$. For each critical point, decide if it is a local max, local min, neither, or if there is not enough information to determine this (NEI). Circle your answers.

Answer: $x = 2.5$ is a: LOCAL MIN $\boxed{\text{LOCAL MAX}}$ NEITHER NEI

Answer: $x = \pi$ is a: $\boxed{\text{LOCAL MIN}}$ LOCAL MAX NEITHER NEI

e. [2 points] On which of the following interval(s) *must* $f(x)$ have an inflection point? Circle all correct answers.

$[0,3]$ $[2,3]$ $[3,9]$

Solution: We know that for $f(x)$ to have an inflection point p, the sign of $f''(x)$ must change at p. The sign of $f''(x)$ must change during the interval $[0,3]$, but it does not happen at a single point – for example, this can occur if $f''(x)$ is first positive, and then zero over an interval, and then negative. So the correct answer is that *none* of the intervals must have an inflection point; however full credit was also given if only $[0,3]$ was circled.