

7. [10 points] Zerina owns a small business selling custom screen-printed and embroidered apparel.
- a. Zerina receives orders for embroidered polo shirts, which she sells for \$11 each. The cost, in dollars, for her to complete an order of q embroidered polo shirts is

$$C(q) = \begin{cases} 6q - \frac{1}{8}q^2 + \frac{56}{9} & 0 \leq q \leq 16 \\ \frac{2}{9}q^{3/2} + 10q - 104 & q > 16. \end{cases}$$

Note that $C(q)$ is continuous for all $q \geq 0$.

- i. [1 point] What is the fixed cost, in dollars, of an order of embroidered polo shirts?

Answer: 56/9

- ii. [5 points] Find the quantity q of embroidered polo shirts in an order that would result in the most profit for Zerina. Assume that, because of storage constraints, Zerina cannot accept an order for more than 80 embroidered polo shirts. Use calculus to find and justify your answer, and make sure you provide enough evidence to fully justify your answer.

Solution: We are given $C(q)$, and know that $R(q) = 11q$. Then since $\pi(q) = R(q) - C(q)$, any point at which $MR = MC$ is a critical point of $\pi(q)$. Now $MR(q) = 11$ and

$$MC(q) = \begin{cases} 6 - \frac{1}{4}q & 0 \leq q < 16 \\ \frac{1}{3}q^{1/2} + 10 & q > 16. \end{cases}$$

We set $MR = MC$ in both of these cases:

$$\begin{array}{ll} 6 - \frac{1}{4}q = 11 & \frac{1}{3}q^{1/2} + 10 = 11 \\ -\frac{1}{4}q = 5 & q^{1/2} = 3 \\ q = -20 & q = 9 \end{array}$$

but neither critical point falls within the domain of the appropriate formula. So there are no points at which $MR = MC$. However, MC is undefined at $q = 16$, since if we plug 16 in to both pieces of $MC(q)$ we get different values. Therefore $\pi'(q)$ is also undefined at $q = 16$. This is the only critical point.

So the possible locations for the global maximum are the endpoints 0 and 80 and the critical point 16. Since $\pi(0) = -56/9$, $\pi(80) \approx 25$, and $\pi(16) \approx 105.7$, an order of 16 polo shirts would result in the most profit for Zerina.

Answer: $q =$ 16

- b. [3 points] Zerina also receives orders for screen-printed t-shirts. When a customer places such an order, they pay a \$6 setup fee, plus \$9 per t-shirt for the first 20 t-shirts ordered. Any additional t-shirts ordered only cost \$7 per t-shirt. Let $P(s)$ be the total price, in dollars, a customer pays for an order of s screen-printed t-shirts. Find a formula for $P(s)$.

Answer: $P(s) = \begin{cases} 6 + 9s & \text{if } 0 \leq s \leq 20 \\ 186 + 7(s - 20) & \text{if } s > 20 \end{cases}$