**4.** [11 points]  A stalagmite is a rock formation that rises from the floor of a cave, while a stalactite is a rock formation that hangs from the cave's ceiling. Throughout this problem, be sure your work is clear.

   **a.** [6 points] A certain stalagmite, in the shape of a cone as shown below at left, is growing both in radius and in height. When the volume of the stalagmite is $36\pi$ cubic inches, how fast is the volume of the stalagmite growing, in cubic inches per year, if at that time the radius is growing by $\frac{1}{200}$ inches per year, the height is 12 inches, and the height is growing at a rate of $\frac{1}{500}$ inches per year?

   *Note that the volume of a cone with radius r and height h is $\frac{1}{3}\pi r^2 h$.*

   **b.** [5 points] A certain stalactite, also in the shape of a cone, has a fixed radius of 3 inches, as shown below at right, but its height is growing. How fast is the height of the stalactite growing when the height is 18 inches, if at this time the area of the sides of the stalactite (not including the circular base) is growing at a rate of 0.2 square inches per year? Include units.

   *Note that the area of the sides (not including the circular base) of a cone of radius r and height h is $\pi r\sqrt{h^2 + r^2}$.*
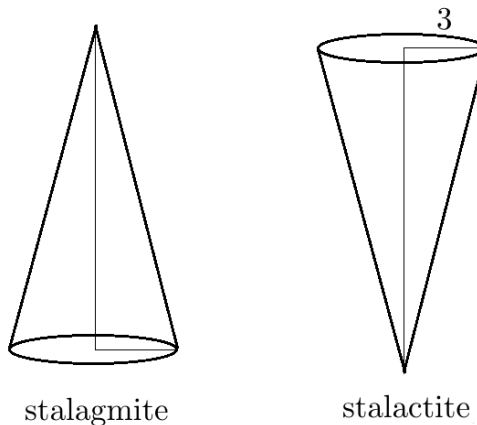


stalagmite          stalactite

**5.** [12 points]  Consider the continuous function

$$g(x) = \begin{cases} -(x^2 + 12x + 37)e^{-x} + 17 & x \le 0 \\ 7x^3 - 21x^2 - 168x - 20 & x > 0. \end{cases}$$

Note that

$$g'(x) = \begin{cases} (x+5)^2 e^{-x} & x < 0 \\ 21(x-4)(x+2) & x > 0. \end{cases}$$

   **a.** [3 points] Find the critical points of $g(x)$.

   **b.** [4 points] Find the $x$-coordinate of all local extrema of $g(x)$, and classify each as a local maximum or a local minimum. Use calculus to find and justify your answers, and be sure to show enough evidence that you have found all local extrema.

   **c.** [5 points] Find the $x$-coordinate of all global extrema of $g(x)$ on the interval $(-\infty, 8]$, and classify each as a global maximum or a global minimum. Use calculus to find your answers, and be sure to show enough evidence to fully justify your answers.