4. [11 points] A stalagmite is a rock formation that rises from the floor of a cave, while a stalactite is a rock formation that hangs from the cave's ceiling. Throughout this problem, be sure your work is clear.

a. [6 points] A certain stalagmite, in the shape of a cone as shown below at left, is growing both in radius and in height. When the volume of the stalagmite is 36π cubic inches, how fast is the volume of the stalagmite growing, in cubic inches per year, if at that time the radius is growing by $\frac{1}{200}$ inches per year, the height is 12 inches, and the height is growing at a rate of $\frac{1}{500}$ inches per year?

Note that the volume of a cone with radius r and height h is $\frac{1}{3}\pi r^2 h$.

Solution: First, note that since $V = \frac{1}{3}\pi r^2 h$, we have that $36\pi = \frac{1}{3}\pi r^2 12$. Thus $r = 3$. Now taking the derivative of the volume equation with respect to time, we have

$$V' = \frac{1}{3}\pi(2rr'h + r^2h').$$

Plugging in our values of r, r', h, and h', we have $V' = \frac{1}{3}\pi(2 * 3 * \frac{1}{200} * 12 + 9 * \frac{1}{500}) = \frac{63\pi}{500} \approx 0.396$ cubic inches per year.

b. [5 points] A certain stalactite, also in the shape of a cone, has a fixed radius of 3 inches, as shown below at right, but its height is growing. How fast is the height of the stalactite growing when the height is 18 inches, if at this time the area of the sides of the stalactite (not including the circular base) is growing at a rate of 0.2 square inches per year? Include units.

Note that the area of the sides (not including the circular base) of a cone of radius r and height h is $\pi r\sqrt{h^2 + r^2}$.

Solution: We have $A = \pi r\sqrt{h^2 + r^2} = 3\pi\sqrt{h^2 + 9}$ since the height is constant. Taking the derivative of both sides with respect to time, we have $A' = \dfrac{3\pi(2hh')}{2\sqrt{h^2+9}}$. Using the given values of A' and h, we have

$$0.2 = \frac{6 * 18\pi h'}{2\sqrt{18^2 + 9}} = \frac{108\pi h'}{2\sqrt{333}}.$$

Thus $h' = \dfrac{0.4 * \sqrt{333}}{108\pi} \approx 0.022$ inches per year.