5. [12 points] Consider the continuous function

$$g(x) = \begin{cases} -(x^2+12x+37)e^{-x}+17 & x \le 0 \\ 7x^3-21x^2-168x-20 & x > 0. \end{cases}$$

Note that

$$g'(x) = \begin{cases} (x+5)^2e^{-x} & x < 0 \\ 21(x-4)(x+2) & x > 0. \end{cases}$$

a. [3 points] Find the critical points of $g(x)$.

Solution: The derivative is zero at $x=-5$ and 4. The derivative is undefined at $x=0$, as the derivative approaches 25 to the left of zero but -168 to the right of zero.

b. [4 points] Find the x-coordinate of all local extrema of $g(x)$, and classify each as a local maximum or a local minimum. Use calculus to find and justify your answers, and be sure to show enough evidence that you have found all local extrema.

Solution:
We will use the first derivative test.

	$x<-5$	$-5<x<0$	$0<x<4$	$4<x$
$g'(x)$	$+\cdot+=+$	$+\cdot+=+$	$-\cdot+=-$	$+\cdot+=+$

Thus we have a local max at $x=0$ and a local min at $x=4$.

c. [5 points] Find the x-coordinate of all global extrema of $g(x)$ on the interval $(-\infty, 8]$, and classify each as a global maximum or a global minimum. Use calculus to find your answers, and be sure to show enough evidence to fully justify your answers.

Solution: We check the value of $g(x)$ at its critical points as well as the end behavior.

$\lim_{x\to-\infty} g(x) = -\infty$

$g(-5) = -2e^5+17 \approx -279.826$

$g(0) = -20$

$g(4) = -580$

$g(8) = 876$

Thus there is no global min, and the global max is at $x=8$.