7. [13 points] Suppose that $f(t)$ is a differentiable, increasing function defined for all real numbers. Some values of $f(t)$ are listed in the table below. Assume that $f'(t)$ is continuous.

t	2.5	3.1	4.0	4.5	5.5	7.0	8.5	9.4	10	10.5	12.5
$f(t)$	3.2	4.5	6.5	7.2	8.5	9.2	9.8	10.5	11.2	12.5	14.5

a. [2 points] Compute $\displaystyle\int_4^7 f'(t)dt$ **exactly**, or write NEI if there is not enough information to do so.

Solution: $\displaystyle\int_4^7 f'(t)dt = f(7) - f(4) = 9.2 - 6.5 = 2.7$

b. [2 points] Compute the average value of $f'(t)$ on the interval $[4.5, 10]$ **exactly**, or write NEI if there is not enough information to do so.

Solution: $\displaystyle\frac{1}{10-4.5}\int_{4.5}^{10} f'(t)dt = \frac{f(10)-f(4.5)}{10-4.5} = \frac{11.2-7.2}{5.5} \approx 0.727.$

c. [2 points] Estimate $f'(9)$.

Solution: $\displaystyle\frac{f(9.4)-f(8.5)}{9.4-8.5} = \frac{10.5-9.8}{0.9} = \frac{7}{9} \approx 0.78$

d. [2 points] Use a left-hand Riemann sum with five equal subdivisions to estimate $\displaystyle\int_{2.5}^{10} f(t)dt$. Write out all the terms in your sum.

Solution: $1.5(f(2.5)+f(4)+f(5.5)+f(7)+f(8.5)) = 1.5(3.2+6.5+8.5+9.2+9.8) = 1.5(37.2) = 55.8$

e. [2 points] Does your answer to part **d.** overestimate, underestimate, or equal the value of $\displaystyle\int_{2.5}^{10} f(t)dt$? Explain your answer.

Solution: $f(t)$ is increasing, so the left-hand sum is an underestimate.

f. [3 points] Use a right-hand Riemann sum with four equal subdivisions to estimate $\displaystyle\int_{4.5}^{12.5} f^{-1}(t)dt$. Write out all the terms in your sum.

Solution: $2(f^{-1}(6.5)+f^{-1}(8.5)+f^{-1}(10.5)+f^{-1}(12.5)) = 2(4+5.5+9.4+10.5) = 2(29.4) = 58.8$