8. [7 points] Gretchen wants to build a rectangular garden that includes a well on her property. The well sits on a 12 foot by 6 foot concrete base. Gretchen’s plans for the garden, which will have length \(a\) feet and width \(b\) feet, are as shown.

Gretchen plans to build a rectangular fence around her entire garden, including on the two outside edges of the well. In addition, she wants the usable area of the garden, that is, the area in the garden other than the base of the well, to be 600 square feet.

a. [2 points] Find a formula for \(a\) in terms of \(b\).

\[ \text{Solution:} \quad \text{The usable area of the garden is given by } 600 = ab - 72. \quad \text{Thus } a = \frac{672}{b} \]

b. [2 points] Find a formula for \(F(b)\), the amount of fence, in feet, that Gretchen needs to build her garden. Your formula should be in terms of \(b\) only.

\[ \text{Solution:} \quad \text{The amount of fence used is equal to the perimeter, or } 2a + 2b. \quad \text{Substituting, we have } F(b) = 2(\frac{672}{b}) + 2b = \frac{1344}{b} + 2b. \]

c. [3 points] What is the domain of the function \(F(b)\)?

\[ \text{Solution:} \quad \text{We need } a > 12 \text{ and } b > 6. \quad \text{When } a = 12, \text{ our formula from the first part tells us that } b = \frac{672}{12} = 56. \quad \text{Thus the domain is } (6, 56). \]