1. [15 points] Given below is a table of values for a function $T(h)$ and its derivative $T'(h)$. Suppose the functions $T(h)$, $T'(h)$, and $T''(h)$ are all defined and continuous on $(-\infty, \infty)$.

h	-4	-2	0	2	4	6	8
$T(h)$	-1	3	5	6	2	1	0
$T'(h)$	3	2	1	0	-1	0	-2

Assume that between consecutive values of h given in the table above, $T(h)$ is either **always increasing** or **always decreasing**.

In **a.**–**c.**, give numerical answers. *You do not need to show work on this page, but limited partial credit may be awarded for work shown.*

a. [2 points] Find the average rate of change of $T(h)$ on the interval $[-4, 4]$.

Solution:

$$\frac{T(4) - T(-4)}{4 - (-4)} = \frac{2 - (-1)}{4 - (-4)} = \frac{3}{8}.$$

Answer: 3/8

b. [2 points] Use the table to estimate $T''(7)$.

Solution:

$$T''(7) \approx \frac{T'(8) - T'(6)}{8 - 6} = \frac{-2 - 0}{2} = -1.$$

Answer: -1

c. [3 points] Find $\displaystyle\int_0^6 \big(2T'(h) + 3\big)\, dh$.

Solution:

$$\int_0^6 (2T'(h)+3)\, dh = 2\int_0^6 T'(h)\, dh + \int_0^6 3\, dh = 2\big(T(6) - T(0)\big) + 6\cdot 3 = 2(1-5)+18 = 10.$$

Answer: 10

d. [2 points] Find an equation of the tangent line to the graph of $T(h)$ at $h = -4$.

Solution: Using y as the dependent variable and h as the independent variable, an equation of the line tangent to $y = T(h)$ at $h = -4$ is

$$y = T(-4) + T'(-4)(h+4) = -1 + 3(h+4).$$

Answer: $y = -1 + 3(h+4) = 3h + 11$

This problem continues from the previous page. The problem statement is repeated for convenience.

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$T(h)$	-1	3	5	6	2	1	0
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Assume that between consecutive values of h given in the table above, $T(h)$ is either **always increasing** or **always decreasing**.

e. [2 points] Use a right–hand Riemann sum with three equal subdivisions to estimate $\int_{-4}^{2} T(h)\,dh$. Write out all the terms in your sum, which you do not need to simplify.

Solution:

$$2\big(T(-2)+T(0)+T(2)\big) = 2(3+5+6) = 28.$$

f. [1 point] Does the answer to part **e.** overestimate, underestimate, or equal the value of $\int_{-4}^{2} T(h)\,dh$? Circle your answer. If there is not enough information, circle NEI.

Answer: [OVERESTIMATE] UNDERESTIMATE EQUAL NEI

g. [1 point] How many equal subdivisions of $[-4, 2]$ are needed to guarantee that the difference between the left and right Riemann sum approximations of $\int_{-4}^{2} T(h)\,dh$ is at most 1?

Solution: Letting n be the number of subdivisions needed, we have

$$\Big(T(2)-T(-4)\Big)\left(\frac{2-(-4)}{n}\right) = 1,$$

so $\frac{7\cdot 6}{n} = 1$, or $n = 42$.

Answer: 42

h. [2 points] Find a number L that makes the following statement a correct conclusion of the Mean Value Theorem: *There is a number c between -2 and 4 such that $T'(c) = L$.*

Solution:

$$L = \frac{T(4)-T(-2)}{4-(-2)} = \frac{2-3}{4-(-2)} = \frac{-1}{6}.$$

Answer: $L =$ $-1/6$