3. [11 points] Throughout this problem, let $p(x) = -x^3 + 3x + 2$.

a. [2 points] Find the x-coordinates of all critical points of $p(x)$. If there are none, write NONE.

Solution: Since polynomials are differentiable everywhere, the critical points of $p(x)$ will be exactly the points where $p'(x)$ is equal to zero. Solving

$$p'(x) = -3x^2 + 3 = -3(x+1)(x-1) = 0$$

gives us critical points of $x = 1$ and $x = -1$.

Answer: Critical points at $x =$ ± 1

In parts **b.** – **d.** below, you are asked to find extrema (local or global) of $p(x)$ on a given interval. If there are none of a particular type, write NONE. Use calculus to find your answers, and make sure you show enough evidence to justify your conclusions.

b. [3 points] Find the x-coordinates of all local minimum(s) and local maximum(s) of $p(x)$ on the interval $(-2, 3)$.

Solution: Both critical points we found in (a) belong to the interval $(-2, 3)$, so we must check each to see whether it is a local max or min or neither. We can use either the First or Second Derivative Test.

First Derivative Test: by plugging in test points (e.g., $p'(0) = 3$ and $p(-2) = p(2) = -9$), using sign logic, or simply using the fact that $p'(x)$ is a downward opening parabola with roots at ± 1, we get the following sign chart:

$p'(t)$: $-$ | -1 | $+$ | 1 | $-$

Thus $p'(x)$ changes sign from negative to positive at $x = -1$ and from positive to negative at $x = 1$, making -1 a local min and 1 a local max of $p(x)$.

Second Derivative Test: We have $p''(x) = -6x$, so $p''(-1) = 6 > 0$ and $p''(1) = -6 < 0$. Thus $p(x)$ is concave up near -1 and concave down near 1, so -1 is a local min and 1 a local max of $p(x)$.

Answer: Local min(s) at $x =$ -1

Answer: Local max(es) at $x =$ 1

c. [4 points] Find the x-coordinates of all global minimum(s) and global maximum(s) of $p(x)$ on the interval $[-2, 3]$.

Solution: Since $p(x)$ is continuous, we know that it has both a max and a min on $[-2, 3]$, and these extrema must occur at either a critical point of $p(x)$ or an endpoint of $[-2, 3]$. So we evaluate $p(x)$ at all such points, and look for the greatest and least values. We have:

$$p(-2) = 4, \qquad p(-1) = 0, \qquad p(1) = 4, \qquad p(3) = -16.$$

From this we see that the global minimum of $p(x)$ on $[-2, 3]$ is -16, occurring at $x = 3$, and the global maximum of $p(x)$ on $[-2, 3]$ is 4, occurring at $x = -2$ and $x = 1$.

Answer: Global min(s) at $x =$ 3

Answer: Global max(es) at $x =$ -2 **and** 1

d. [2 points] Find the x-coordinates of all global minimum(s) and global maximum(s) of $p(x)$ on the interval $(-2, 3)$.

Solution: Based on our results from part (c), the global maximum of $p(x)$ on $(-2, 3)$ will be 4, occurring just at $x = 1$ this time since we are excluding the endpoint $x = -2$ from our interval. Furthermore, the values of $p(x)$ will approach -16 as x tends to the right endpoint $x = 3$, but since we are now excluding this endpoint from our interval, $p(x)$ will never actually reach -16, so $p(x)$ has no global minimum on $(-2, 3)$.

Answer: Global min(s) at $x =$ **none**

Answer: Global max(es) at $x =$ 1