6. [8 points] In this problem, you may use the following facts:

- The surface area S of a sphere of radius r is given by $S = 4\pi r^2$.
- The volume V of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.

Suppose a spherical snowball is melting so that its surface area decreases at the constant rate of 20 cm^2 per minute. Use this information to answer the following questions, and remember to include appropriate units in your answers.

a. [4 points] How fast is the radius of the snowball changing when the radius is 5 cm?

Solution: Let r be the radius, S the surface area, and V the volume of our melting snowball, so each of r, S, and V is a function of time, t, where t is measured in minutes and r, S, and V are measured in cm, cm^2, and cm^3, respectively.

We want to find $\frac{dr}{dt}$ when $r = 5$. We are given that $\frac{dS}{dt} = -20$. Differentiating the equation $S = 4\pi r^2$ with respect to t, we obtain

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}.$$

Substituting in $\frac{dS}{dt} = -20$ and $r = 5$ and solving for $\frac{dr}{dt}$ gives us

$$\frac{dr}{dt} = \frac{1}{8\pi \cdot 5} \cdot (-20) = -\frac{1}{2\pi} \text{ cm/min}.$$

Answer: $-\frac{1}{2\pi}$ **cm/min**

b. [4 points] How fast is the volume of the snowball changing when the radius is 5 cm?

Solution: Now we want to find $\frac{dV}{dt}$ when $r = 5$. Differentiating the equation $V = \frac{4}{3}\pi r^3$ with respect to t, we get

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}.$$

Substituting in $r = 5$ and $\frac{dr}{dt} = -\frac{1}{2\pi}$ from part (a), we get

$$\frac{dV}{dt} = 4\pi(5^2) \cdot \frac{-1}{2\pi} = -50 \text{ cm}^3/\text{min}.$$

Answer: -50 $\mathbf{cm^3/min}$