1. [13 points] Given below is a table of values for a function $f(x)$ and its derivative $f'(x)$. The functions $f(x)$, $f'(x)$, and $f''(x)$ are all defined and continuous on $(-\infty, \infty)$.

x	0	2	4	6	8	10	12
$f(x)$	15	12	11	7	-2	3	5
$f'(x)$	-3	0	-2	-4	0	2	6

Assume that between consecutive values of x given in the table above, $f(x)$ is either **always increasing** or **always decreasing**.

In **a.**–**d.**, find the numerical value **exactly**, or write NEI if there is not enough information provided to do so. You do not need to simplify your numerical answers. *You do not need to show work on this page, but limited partial credit may be awarded for work shown.*

a. [2 points] Find $\lim_{s\to 0} \dfrac{f(4+s)-f(4)}{s}$.

Solution: By the definition of derivative, $\lim_{s\to 0} \dfrac{f(4+s)-f(4)}{s} = f'(4) = -2$.

Answer: -2

b. [2 points] If $B(x) = x^3 f(x)$, find $B'(10)$.

Solution: By the Product Rule, $B'(x) = 3x^2 f(x) + x^3 f'(x)$, so

$$B'(10) = 3 \cdot 10^2 \cdot f(10) + 10^3 \cdot f'(10) \;=\; 300 \cdot 3 + 1000 \cdot 2 \;=\; 2900.$$

Answer: 2900

c. [2 points] Find $\displaystyle\int_4^8 f'(x)\, dx$.

Solution: By the Fundamental Theorem of Calculus,

$$\int_4^8 f'(x)\, dx \;=\; f(8) - f(4) \;=\; -2 - 11 \;=\; -13.$$

Answer: -13

d. [2 points] Find $\displaystyle\int_0^{10} \left(5f''(x) - 3x^2\right)\, dx$.

Solution: By properties of integrals and the Fundamental Theorem of Calculus,

$$\begin{aligned}\int_0^{10} \left(5f''(x) - 3x^2\right)\, dx \;&=\; 5\int_0^{10} f''(x)\, dx - 3\int_0^{10} x^2\, dx \\ &=\; 5\big(f'(10) - f'(0)\big) - 3\left(\tfrac{1}{3}x^3\big]_0^{10}\right) \\ &=\; 5\big(2-(-3)\big) - 3\big(1000/3\big) \;=\; 25 - 1000 \;=\; -975.\end{aligned}$$

Answer: -975

This problem continues on the next page.

This problem continues from the previous page. The problem statement is repeated for convenience.

Given below is a table of values for a function $f(x)$ and its derivative $f'(x)$. The functions $f(x)$, $f'(x)$, and $f''(x)$ are all defined and continuous on $(-\infty, \infty)$.

x	0	2	4	6	8	10	12
$f(x)$	15	12	11	7	-2	3	5
$f'(x)$	-3	0	-2	-4	0	2	6

Assume that between consecutive values of x given in the table above, $f(x)$ is either **always increasing** or **always decreasing**.

e. [2 points] Use a right Riemann sum with four equal subdivisions to estimate $\displaystyle\int_0^8 f(x)\,dx$. Write out all the terms in your sum. Your answer should not include the letter f, but you do not need to simplify.

Solution:

$$2\Big(f(2)+f(4)+f(6)+f(8)\Big) \;=\; 2\big(12+11+7-2\big) \;=\; 56.$$

f. [1 point] Does the answer to part **e.** overestimate, underestimate, or equal the value of $\displaystyle\int_0^8 f(x)\,dx$? Circle your answer. If there is not enough information to decide, circle NEI.

Answer: OVERESTIMATE UNDERESTIMATE EQUAL NEI

g. [2 points] Use a left Riemann sum with two equal subdivisions to estimate $\displaystyle\int_2^8 x f(2x)\,dx$. Write out all the terms in your sum. Your answer should not include the letter f, but you do not need to simplify.

Solution:

$$3\Big(2\cdot f(4)+5\cdot f(10)\Big) \;=\; 3\cdot\big(2\cdot 11+5\cdot 3\big) \;=\; 3\cdot 37 \;=\; 111.$$