5. [11 points]

Two cars, A and B, are driving along straight roads towards a common intersection, as pictured to the right. Car A is driving due east, and Car B due north. At 12pm, Car A is west of the intersection and Car B south of it, so that they will both reach the intersection sometime later that afternoon.

Let u(t) be the instantaneous velocity of Car A t hours after 12pm, and v(t) the instantaneous velocity of Car B t hours after 12pm, both in kilometers per hour (kph). Assume u(t) is positive when Car A is traveling east, and v(t) is positive when Car B is traveling north.

a. [2 points] Assume Car A reaches the intersection a hours after 12pm, and Car B reaches the intersection b hours after 12pm. Circle the <u>one best</u> practical interpretation of the equation

$$\int_0^a u(t) \, dt = \int_0^b v(t) \, dt.$$

- *i*. The two cars arrive at the intersection at the same time.
- *ii.* The two cars pass through the intersection traveling the same speed.
- *iii.* The two cars start the same distance away from the intersection at 12pm.
- *iv.* The difference between the two cars' initial speeds at 12pm equals the difference between their speeds at the moments when they pass through the intersection.
- v. NONE OF THESE.
- **b**. [3 points] Circle the <u>one</u> equation below that **best** represents the following statement: "Car A is traveling about 5 kph slower at 2:03pm than it was traveling at 2pm."
 - (i) $(u^{-1})'(5) = \frac{3}{60}$ (iii) u'(2.05) = -100 (v) $\int_2^{2.03} u(t) dt = -5/3$
 - (ii) $(u^{-1})'(5) = 2$ (iv) u'(2) = -5/3 (vi) $\int_2^{2.03} u'(t) dt = -5/3$
- c. [3 points] Write a sentence that gives a practical interpretation of the equation

$$\int_0^3 u(t) \, dt = 296.$$

Solution: At 3pm, Car A's position is 296km to the east of its position at 12pm.

d. [3 points] Write a mathematical equation that represents the following statement: "The average velocity of Car B over the three hours between 12pm and 3pm is 99 kph."

©2023 Univ of Michigan Dept of Mathematics Creative Commons BY-NC-SA 4.0 International License

Answer:
$$\frac{1}{3}\int_0^3 v(t) dt$$

= 99

$$\begin{array}{c} y \\ \hline \\ A \\ \hline \\ B \\ \hline \\ \end{array} \\ \end{array} x$$