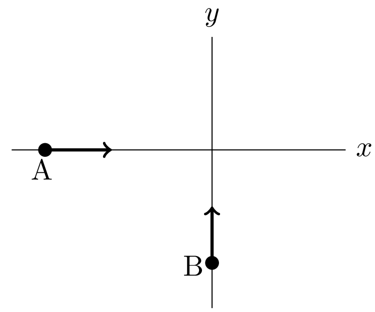
6. [6 points]

Assume the same setup in this problem as in the previous problem, and now additionally assume that at 1pm, Car A is 8 km west of the intersection traveling east at 50 kph, while Car B is 6 km south of the intersection traveling north at 100 kph. How fast is the distance between the two cars changing at 1pm?



Solution: Let us write x for the x-coordinate of Car A's location along its road t hours after 12pm, y for the y-coordinate of Car B's location along its road t hours after 12pm, and z for the distance between Cars A and B t hours after 12pm. So $x^2 + y^2 = z^2$ by the Pythagorean Theorem, and we are looking for $\frac{dz}{dt}$ when $t = 1$. Differentiating $x^2 + y^2 = z^2$ with respect to t gives us

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} \;=\; 2z\frac{dz}{dt}. \tag{1}$$

To find $\frac{dz}{dt}$ when $t = 1$, we plug in $x = -8$, $\frac{dx}{dt} = 50$, $y = -6$, $\frac{dy}{dt} = 100$, and

$$z \;=\; \sqrt{(-8)^2 + (-6)^2} \;=\; \sqrt{100} \;=\; 10$$

into Equation (1) and solve for $\frac{dz}{dt}$ to get:

$$\frac{dz}{dt} \;=\; \frac{2(-8)(50) + 2(-6)(100)}{2 \cdot 10} \;=\; -8 \cdot 5 - 6 \cdot 10 \;=\; -40 - 60 \;=\; -100.$$

This means the distance between the two cars is *decreasing* at a rate of 100 kph at 1pm.

Answer: The distance is INCREASING [DECREASING] at a rate of 100 kph.

7. [6 points] Find all local extrema of the function $p(x) = x^5 - 5x^4 + 5x^3 + 1$, and classify each as a local maximum or a local minimum. If there are none of a particular type, write NONE. Use calculus to find your answers, and make sure you **show enough evidence** to justify your conclusions.

Solution: We need to find the critical points of $p(x)$ and test them to determine which ones are local extrema. Differentiating $p(x)$ and factoring, we get

$$p'(x) \;=\; 5x^4 - 20x^3 + 15x^2 \;=\; 5x^2(x^2 - 4x + 3) \;=\; 5x^2(x-3)(x-1).$$

The polynomial function $p(x)$ is differentiable everywhere, so its only critical points are the roots of $p'(x)$, which are $x = 0, 1, 3$. Now we use sign logic to find the sign of $p'(x) = 5x^2(x-3)(x-1)$ in each interval determined by these critical points:

	$x<0$	0	$0<x<1$	1	$1<x<3$	3	$x>3$
$p'(x)$:	$+$		$+$		$-$		$+$
	$+\,-\,-$		$+\,-\,-$		$+\,-\,+$		$+\,+\,+$

Alternatively, we could find these signs by plugging test points from each interval into $p'(x)$, or by sketching a graph of $p'(x)$. However we determine them, it follows from the First Derivative Test that $p(x)$ has a local min at $x = 3$ and a local max at $x = 1$, while $x = 0$ is not a local extremum of $p(x)$ since $p'(x)$ does not change sign at $x = 0$.

Answer: Local min(s) at $x =$ 3

Answer: Local max(es) at $x =$ 1