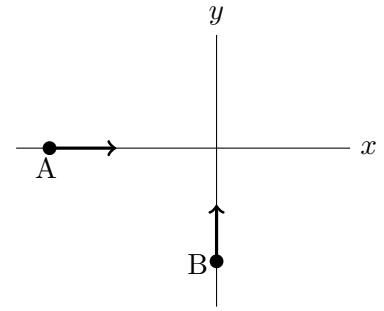


6. [6 points]

Assume the same setup in this problem as in the previous problem, and now additionally assume that at 1pm, Car A is 8 km west of the intersection traveling east at 50 kph, while Car B is 6 km south of the intersection traveling north at 100 kph. How fast is the distance between the two cars changing at 1pm?



Solution: Let us write x for the x -coordinate of Car A's location along its road t hours after 12pm, y for the y -coordinate of Car B's location along its road t hours after 12pm, and z for the distance between Cars A and B t hours after 12pm. So $x^2 + y^2 = z^2$ by the Pythagorean Theorem, and we are looking for $\frac{dz}{dt}$ when $t = 1$. Differentiating $x^2 + y^2 = z^2$ with respect to t gives us

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}. \quad (1)$$

To find $\frac{dz}{dt}$ when $t = 1$, we plug in $x = -8$, $\frac{dx}{dt} = 50$, $y = -6$, $\frac{dy}{dt} = 100$, and

$$z = \sqrt{(-8)^2 + (-6)^2} = \sqrt{100} = 10$$

into Equation (1) and solve for $\frac{dz}{dt}$ to get:

$$\frac{dz}{dt} = \frac{2(-8)(50) + 2(-6)(100)}{2 \cdot 10} = -8 \cdot 5 - 6 \cdot 10 = -40 - 60 = -100.$$

This means the distance between the two cars is *decreasing* at a rate of 100 kph at 1pm.

Answer: The distance is INCREASING DECREASING at a rate of 100 kph.

7. [6 points] Find all local extrema of the function $p(x) = x^5 - 5x^4 + 5x^3 + 1$, and classify each as a local maximum or a local minimum. If there are none of a particular type, write NONE. Use calculus to find your answers, and make sure you **show enough evidence** to justify your conclusions.

Solution: We need to find the critical points of $p(x)$ and test them to determine which ones are local extrema. Differentiating $p(x)$ and factoring, we get

$$p'(x) = 5x^4 - 20x^3 + 15x^2 = 5x^2(x^2 - 4x + 3) = 5x^2(x - 3)(x - 1).$$

The polynomial function $p(x)$ is differentiable everywhere, so its only critical points are the roots of $p'(x)$, which are $x = 0, 1, 3$. Now we use sign logic to find the sign of $p'(x) = 5x^2(x - 3)(x - 1)$ in each interval determined by these critical points:

$$p'(x): \begin{array}{ccccccc} & + & & + & & - & & + \\ \hline & & 0 & & 1 & & 3 & \\ + & - & - & 0 & + & - & + & + & + & + \end{array}$$

Alternatively, we could find these signs by plugging test points from each interval into $p'(x)$, or by sketching a graph of $p'(x)$. However we determine them, it follows from the First Derivative Test that $p(x)$ has a local min at $x = 3$ and a local max at $x = 1$, while $x = 0$ is not a local extremum of $p(x)$ since $p'(x)$ does not change sign at $x = 0$.

Answer: Local min(s) at $x =$ 3

Answer: Local max(es) at $x =$ 1