8. [8 points] After its first month of production, Alana's company has manufactured 1.2 million Stan Lee cups. Now she is trying to decide whether it would be in the company's interest to halt production here, or else continue manufacturing more cups.

Suppose $C(q)$ and $R(q)$ are the cost and revenue functions, respectively, of the company producing and selling q million cups, and let $\pi(q) = R(q) - C(q)$ be the profit function. Assume $C(q)$ and $R(q)$ are differentiable for all $q > 0$, and that Alana can sell every cup she produces.

The parts below describe different scenarios that are independent of each other. In each scenario, determine whether Alana should *continue producing more cups* or instead *halt production at 1.2 million cups* in order to maximize profit. If there is not enough information to decide, answer NEI. Circle the one best answer.

a. [1 point] Suppose $MR(1.2) > MC(1.2)$.

$\boxed{\text{PRODUCE MORE CUPS}}$ HALT PRODUCTION AT 1.2 MILLION NEI

b. [1 point] Suppose $C(1.2) > R(1.2)$.

PRODUCE MORE CUPS HALT PRODUCTION AT 1.2 MILLION $\boxed{\text{NEI}}$

c. [2 points] Suppose $\pi'(1.2) = 0$.

PRODUCE MORE CUPS HALT PRODUCTION AT 1.2 MILLION $\boxed{\text{NEI}}$

d. [2 points] Suppose $\pi'(q) < 0$ for all $q > 1.2$.

PRODUCE MORE CUPS $\boxed{\text{HALT PRODUCTION AT 1.2 MILLION}}$ NEI

e. [2 points] Suppose $\pi(q)$ has a local minimum at $q = 1.2$.

$\boxed{\text{PRODUCE MORE CUPS}}$ HALT PRODUCTION AT 1.2 MILLION NEI

9. [11 points] Consider the family of ellipses $\mathcal{C}$ defined implicitly by the equation

$$k^{-4}x^2 + e^{2k}y^2 = 1,$$

where $k > 0$ is a parameter.

a. [2 points] Find the unique value of k such that the point $(0, \frac{1}{2})$ lies on the ellipse $\mathcal{C}$.

Solution: We plug $x = 0$ and $y = \frac{1}{2}$ into the equation that defines $\mathcal{C}$, and solve for k:

$$\begin{aligned} k^{-4} \cdot 0^2 + e^{2k}(1/2)^2 &= 1, \\ e^{2k} &= 4, \\ k &= \frac{1}{2}\ln 4 = \ln 2. \end{aligned}$$

Answer: $k =$ $\ln 2$

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