

10. [10 points] Let  $m(x)$  be a twice-differentiable function that is defined for all real numbers. Suppose the *only* critical point of  $m(x)$  is  $x = 0$ , and that

$$m''(x) = \frac{x^2(9 - x^4)}{(x^4 + 2)^3}.$$

- a. [4 points] Find the intervals of concavity of  $m(x)$ . That is, find the largest open intervals on which  $m(x)$  is concave up, and the largest open intervals on which  $m(x)$  is concave down. *Show enough work to fully justify your conclusions.*

**Answer:** Intervals on which  $m(x)$  is **concave up**: \_\_\_\_\_

**Answer:** Intervals on which  $m(x)$  is **concave down**: \_\_\_\_\_

- b. [1 point] Using your work in part (a), list all inflection points of  $m(x)$ , separated by commas. *No additional justification necessary.*

**Answer:**  $m(x)$  has inflection points at  $x =$  \_\_\_\_\_

- c. [3 points] Using your work above, classify  $x = 0$  as a LOCAL MAX of  $m(x)$ , a LOCAL MIN of  $m(x)$ , or NEITHER by circling your answer below, or else circle NEI if there is not enough information to tell. *Include a brief justification of your answer.*

LOCAL MAX

LOCAL MIN

NEITHER

NEI

- d. [2 points] Find the following limits. Write DNE for any limit that does not exist, even if the limit tends to  $\pm\infty$ .

i.  $\lim_{x \rightarrow \infty} m''(x)$

**Answer:** \_\_\_\_\_

ii.  $\lim_{x \rightarrow 0} \frac{m''(x)}{x^2}$

**Answer:** \_\_\_\_\_