1. [12 points] Given below is a table of values for an **even** function $g(x)$. Assume the function $g(x)$ and its derivative $g'(x)$ are defined and continuous on $(-\infty, \infty)$.

x	-2	0	2	4	6	8	10	12
$g(x)$	2	0	2	5	8	3	2	3

Assume that between consecutive values of x given in the table above, $g(x)$ is either **always increasing** or **always decreasing**.

a. [2 points] Find $\displaystyle\int_2^4 \big(2g'(x) - 3x\big)\,dx$.

Solution:

$$\int_2^4 \big(2g'(x) - 3x\big)\,dx \;=\; 2\int_2^4 g'(x), dx - \int_2^4 3x\,dx \;=\; 2\Big(g(4) - g(2)\Big) - \Big(\frac{3}{2}\cdot 4^2 - \frac{3}{2}\cdot 2^2\Big)$$
$$= \;2(5-2) - \frac{3}{2}(16-4) \;=\; 6 - 18 \;=\; -12.$$

Answer: -12

b. [3 points] Find the average of value of $g(x)$ on the interval $[-5, 5]$ given that $\displaystyle\int_0^5 4g(x)\,dx = 60$.

Solution: Given that $g(x)$ is even and $\int_0^5 4g(x)\,dx = 60$, the average value of $g(x)$ on $[-5, 5]$ is

$$\frac{1}{5-(-5)}\int_{-5}^5 g(x)\,dx \;=\; \frac{2}{10}\int_0^5 g(x)\,dx \;=\; \frac{2}{40}\int_0^5 4g(x)\,dx \;=\; \frac{1}{20}\cdot 60 \;=\; 3.$$

Answer: 3

c. [2 points] Find a number M that makes the following statement a correct conclusion of the Mean Value Theorem: *There is a number c between* 6 *and* 8 *such that $g'(c) = M$.*

Solution: $M = \frac{g(8)-g(6)}{8-6} = \frac{3-8}{2} = -\frac{5}{2}$.

Answer: $M =$ $-5/2$

d. [2 points] Use a right-hand Riemann Sum with 3 equal subdivisions to estimate $\displaystyle\int_0^6 g(x)\,dx$.

Solution: $2\big(g(2) + g(4) + g(6)\big) = 2\big(2 + 5 + 8\big) = 2(15) = 30$.

e. [1 point] Is the estimate in part **d.** an overestimate or an underestimate? Circle your answer below, or circle NEI if there is not enough information to tell.

UNDERESTIMATE | **[OVERESTIMATE]** | NEI

Solution: It is an overestimate since $g(x)$ is increasing on $[0, 6]$.

f. [2 points] How many equal subdivisions of $[0, 6]$ are needed so that the difference between the left-hand and right-hand Riemann sum approximations of $\displaystyle\int_0^6 g(x)\,dx$ is exactly 1?

Solution: If we divide $[0, 6]$ into n subintervals, then $\Delta x = \frac{6-0}{n}$, and the error is $|g(6) - g(0)|\frac{6}{n} = 1$. Solving for n gives $n = 6|g(6) - g(0)| = 6 \cdot 8 = 48$.

Answer: $n =$ 48