2. [10 points] Consider the family of functions $f(x) = x^2e^{ax}$ where $a > 0$. *Show all your work in each part below.*

a. [2 points] Find the unique value of a such that $f(2) = 12$.

Solution: Plugging in $x = 2$, we have

$$12 = f(2) = 2^2 e^{a\cdot 2} = 4e^{2a}, \qquad \text{so} \qquad 3 = e^{2a}.$$

Solving this for a gives us $\ln 3 = 2a$, or $a = \frac{1}{2}\ln 3$.

Answer: $a =$ $\frac{1}{2}\ln 3$

Note: in the parts below, remember that a is a parameter, not the value you just found in part **a**.

b. [2 points] Find the derivative $f'(x)$ in terms of the parameter a.

Solution: Using the Product Rule and keeping in mind that a is a parameter, we have

$$f'(x) = 2xe^{ax} + ax^2e^{ax}.$$

Answer: $f'(x) =$ $2xe^{ax} + ax^2e^{ax}$

c. [2 points] Find all critical points of $f(x)$ in terms of the parameter a.

Solution: Factoring our expression for $f'(x)$ above, we get

$$f'(x) = 2xe^{ax} + ax^2e^{ax} = xe^{ax}\Big(2 + ax\Big).$$

From this we see that $f'(x)$ is defined everywhere, and equals zero when $x = 0$ or $2 + ax = 0$, that is, when $x = 0$ or $x = -\frac{2}{a}$. (Since we know $a > 0$, we do not need to worry about dividing by a.)

Answer: $x =$ $x = 0$ **and** $x = -2/a$

d. [4 points] Find all local extrema of $f(x)$ in terms of a. If there are none of a particular type, write NONE. *Use calculus to find your answers, and show enough evidence to justify them.*

Solution: We apply the First Derivative Test to $f(x)$. From **c**., we have $f'(x) = xe^{ax}(2 + ax)$. Applying sign logic to these three factors, we get:

$f'(x)$:	$x < -2/a$	$-2/a$	$-2/a < x < 0$	0	$x > 0$
	$- \cdot + \cdot - = +$		$- \cdot + \cdot + = -$		$+ \cdot + \cdot + = +$

So f' is positive for $x < -2/a$ and $x > 0$, and negative for $-2/a < x < 0$. This means $x = -2/a$ is a local max of $f(x)$, while $x = 0$ is a local min of $f(x)$, by the First Derivative Test.

Answer: Local min(s) at $x =$ 0 and Local max(es) at $x =$ $-2/a$