4. [11 points] Suppose the rate at which the amount of carbon dioxide (CO_2) in Walden Pond is changing t hours after 6am, in kilograms per hour, is given by the continuous function $h(t)$. Some values of $h(t)$ are given in the table below. Assume that between consecutive values of t given in the table, $h(t)$ is either **always increasing** or **always decreasing**.

t	0	3	6	12	15	18	21
$h(t)$	-2	-5	0	6	8	7	0

No justification is required in any part of this problem, but partial credit may be awarded for work.

a. [2 points] Write an expression involving an integral that represents the change in the amount of CO_2 in Walden pond between 9am and 12 noon.

Answer: $\int_3^6 h(t)\,dt$ kg

b. [2 points] Write an expression involving an integral that represents the average rate of change of CO_2 in Walden pond between 6am and 6pm.

Answer: $\frac{1}{12}\int_0^{12} h(t)\,dt$ kg/hr

c. [7 points] Suppose $H(t)$ is the amount of CO_2 in Walden Pond t hours after 6am, in kilograms, and assume $H(0) = 600$.

i. Put the following quantities in order from *least* to *greatest*.

$H(0)$ $H(3)$ $H(18)$ $H(21)$ $H'(6)$ $h(0)$

Solution: Since $-5 \le h(t) \le -2$ for all $0 \le t \le 3$, we have $600 + 3(-5) \le H(3) < H(0)$. Since $H'(6) = h(6) = 0$, this means $h(0) < H'(6) < H(3) < H(0)$. Also, we have $H(18) < H(21)$ since $h(t) \ge 0$ for all $18 \le t \le 21$. Finally, since $h(t) \ge -5$ for all $0 \le t \le 6$, while $h(t) \ge 0$ for all $6 \le t \le 12$ and $h(t) \ge 6$ for all $12 \le t \le 18$, we have

$$H(18) - H(0) = \int_0^{18} h(t)\,dt \ge 0.$$

Putting all this together gives us $h(0) < H'(6) < H(3) < H(0) < H(18) < H(21)$.

Answer: $h(0)$ < $H'(6)$ < $H(3)$ < $H(0)$ < $H(18)$ < $H(21)$

LEAST GREATEST

ii. Write an expression which does not include a capital "H" that is equal to $H(24)$. You may use the function $h(t)$, along with any integrals, derivatives, or numbers that you want.

Answer: $H(24) = 600 + \int_0^{24} h(t)\,dt$