6. [7 points] Continue to assume the setup of the previous problem, so Ivan is $x = f(t)$ meters east of his starting point t seconds after 12 noon, walking back and forth along a straight line. Suppose also that Opal is driving in circles around Ivan and blasting her car stereo, so that:

- the distance r, in meters, between Opal and Ivan t seconds after 12 noon is given by the function $r = g(t)$;
- when Ivan is r meters from Opal, the loudness of Opal's stereo in decibels as perceived by Ivan is given by $L(r) = 100 - 20\log(r)$. [Recall that "log" means log base 10.]

a. [2 points] Find $L'(10)$.

Solution: Using the rule for differentiating logarithmic functions, we have

$$L'(r) = -20 \cdot \frac{1}{(\ln 10)r},$$

so

$$L'(10) = \frac{-2}{\ln 10}.$$

Answer: $L'(10) =$ $-2/\ln(10)$

b. [5 points] At what rate is the loudness of Opal's stereo, as perceived by Ivan, changing with respect to time when Ivan is 10 meters from Opal and moving away from her at a speed of 2 meters per second? *Include units.*

Solution: The loudness of Opal's stereo in decibels, as perceived by Ivan t seconds after noon, is given by the composition $L(g(t))$. To find the rate at which this is changing at a given time t, we differentiate using the Chain Rule to obtain $L'(g(t))g'(t)$. When Ivan and Opal are 10 meters apart and moving away from each other at a speed of 2 meters per second, we have $g(t) = 10$ and $g'(t) = 2$, so at this moment

$$L'(g(t))g'(t) = L'(10) \cdot 2 = -4/\ln(10) \quad \text{decibels per second.}$$

Solution: Alternatively, we can solve this using Leibniz notation. Let us write $y = L(r)$ for the loudness of Opal's stereo, as perceived by Ivan, when they are r meters away from each other. We want to find $\frac{dy}{dt}$ when $r = 10$ and $\frac{dr}{dt} = 2$. Using the Chain Rule, we have

$$\frac{dy}{dt} = \frac{dy}{dr} \cdot \frac{dr}{dt},$$

so when $r = 10$ and $\frac{dr}{dt} = 2$ we have

$$\frac{dy}{dt} = \frac{-2}{\ln 10} \cdot 2 = -4/\ln(10) \quad \text{decibels per second,}$$

since we know from part **a.** that $\left.\frac{dy}{dr}\right|_{r=10} = L'(10) = -2/\ln(10)$.

Answer: $-4/\ln(10)$ **decibels per second**