9. [9 points] Roger owns a second farm of 500 trees which produces both timber and wild honey. He plans to harvest q trees for timber in the month of May, while using the remaining $500 - q$ trees for honey production. The cost in dollars for Roger to run his farm in May if he harvests q trees for timber is given by

$$C(q) = 100 + 5q + \frac{q^2}{5}.$$

a. [2 points] During May, each tree cut for timber generates \$85 in revenue, while each tree used for honey production yields \$20. Write an expression for the revenue $R(q)$ that Roger earns in May for harvesting q trees while using the remaining $500 - q$ trees for honey production.

Solution: During May, Roger earns $85q$ dollars from harvesting q trees, and $20(500-q)$ dollars from producing honey from the remaining $500 - q$ trees, so his total revenue is

$$85q + 20(500 - q) \;=\; 65q + 10{,}000 \quad \text{dollars.}$$

Answer: $R(q) =$ $65q + 10{,}000$ dollars

b. [2 points] Find the marginal revenue $\mathrm{MR}(q)$ and the marginal cost $\mathrm{MC}(q)$ of Roger's operation in the month of May.

Solution: Since marginal revenue and marginal costs are the derivatives of revenue and cost, respectively, we get

$$\mathrm{MR}(q) \;=\; R'(q) \;=\; 85 \qquad \text{and} \qquad \mathrm{MC}(q) \;=\; C'(q) \;=\; 5 + \frac{2}{5}q.$$

$\mathrm{MR}(q) =$ 65, and $\mathrm{MC}(q) =$ $5 + \frac{2}{5}q$

c. [3 points] How many trees should Roger allocate to timber production in order to maximize his May profits? *Use calculus, and show your work. You do not need to fully justify your answer, but partial credit may be awarded for work shown.*

Solution: The derivative of profit is $\mathrm{MR}(q) - \mathrm{MC}(q) = 65 - \left(5 + \frac{2}{5}q\right) = 60 - \frac{2}{5}q$, which equals zero when $q = 150$. To check that this sole critical point $q = 150$ of the profit function is indeed a max, we can apply the First Derivative Test and note that $60 - \frac{2}{5}q > 0$ when $0 \le q < 150$ whle $60 - \frac{2}{5}q < 0$ when $150 < q \le 500$. Alternatively, we could apply the Second Derivative Test and note that $-\frac{2}{5} < 0$, so the profit function is concave down near $q = 150$, hence $q = 150$ is a max.

Answer: $q =$ 150 trees

d. [2 points] Suddenly Roger remembers his sustainability pledge to replant exactly as many trees as he cuts down. If the cost of replanting a single harvested tree is \$$b$, find the *smallest* value of b for which dedicating the entire farm to honey production is at least as profitable as producing both timber and honey.

Solution: If Roger must spend \$$b$ to replant every tree he harvests, then his new marginal cost function is $\mathrm{MC}(q) = 5 = \frac{2}{5}q + b$. Marginal revenue is still 65, and we saw in part **c**. that profit is maximized when MR = MC, that is, when

$$5 + \frac{2}{5}q + b = 65, \qquad \text{or} \qquad \frac{2}{5}q + b = 60.$$

Dedicating the entire farm to honey production means $q = 0$, so plugging $q = 0$ into the equation above and solving for b gives us $b = 60$.

Answer: $b =$ 60