1. [11 points]
Some values of the increasing, differentiable function $h(x)$ and its derivative $h'(x)$ are given in the table to the right.

x	-1	1	3	5
$h(x)$	-3	-2	1	3
$h'(x)$	$\frac{1}{2}$	1	2	$\frac{1}{3}$

a. [2 points] Let $W(x) = h\left(\frac{x}{2}\right)\ln(x)$. Find $W'(6)$.

Solution: By the Product Rule and Chain Rule, we have $W'(x) = \frac{1}{2}h'\left(\frac{x}{2}\right)\ln(x) + h\left(\frac{x}{2}\right)/x$, so

$$W'(6) \;=\; \frac{1}{2}h'(3)\ln(6) + \frac{h(3)}{6} \;=\; \frac{1}{2}\cdot 2\ln(6) + \frac{1}{6} \;=\; \ln(6) + \frac{1}{6}.$$

Answer: $\ln(6) + \dfrac{1}{6}$

b. [2 points] Find the average value of $h'(x)$ on the interval $[-1, 5]$.

Solution: Using the Net Change Theorem, the average value of $h'(x)$ on the interval $[-1, 5]$ is

$$\frac{1}{5-(-1)}\int_{-1}^{5} h'(x)\,dx \;=\; \frac{1}{6}\Big(h(5) - h(-1)\Big) \;=\; \frac{3-(-3)}{6} \;=\; \frac{6}{6} \;=\; 1.$$

Answer: 1

c. [3 points] Find $\displaystyle\int_1^3 \big(4h''(x) + \cos(x)\big)\,dx$.

Solution: Using the Net Change Theorem, we have

$$\int_1^3 \big(4h''(x) + \cos(x)\big)\,dx \;=\; 4\int_1^3 h''(x)\,dx + \int_1^3 \cos(x)\,dx \;=\; 4\Big(h'(3) - h'(1)\Big) + \Big(\sin(3) - \sin(1)\Big)$$
$$= \; 4(2-1) + \sin(3) - \sin(1) \;=\; 4 + \sin(3) - \sin(1).$$

Answer: $4 + \sin(3) - \sin(1)$

d. [2 points] Use a right-hand Riemann sum with two equal subdivisions to estimate $\displaystyle\int_1^5 h(x)\,dx$. Write out all the terms in your sum, which you do not need to simplify.

Solution: $h(3)\cdot 2 + h(5)\cdot 2 \;=\; 1\cdot 2 + 3\cdot 2 \;=\; 8.$

e. [2 points] How many equal subdivisions of $[1, 5]$ are needed so that the difference between the left and right Riemann sum approximations of $\displaystyle\int_1^5 h(x)\,dx$ is exactly 1?

Solution: If we use n equal subdivisions, then each subinterval has width $\Delta x = \frac{5-1}{n} = \frac{4}{n}$, which makes the error between the left and right Riemann sums

$$\big(h(5) - h(1)\big)\Delta x \;=\; \big(3-(-2)\big)\frac{4}{n} \;=\; \frac{20}{n}.$$

Setting this error equal to 1 and solving for n gives us $n = 20$.

Answer: 20