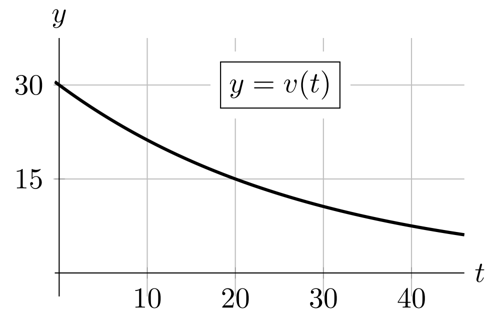
11. [8 points]



A car runs out of gas and begins coasting on a highway. A graph of the car's velocity $v(t)$ in meters per second t seconds after it begins coasting is shown to the right.

Let $d(t)$ be the distance the car has traveled, in meters, since it started coasting, and let $L(t)$ be the linear approximation of $d(t)$ at $t = 20$.

a. [1 point] Approximately how far, in meters, did the car coast over the first 20 seconds after it began coasting? Give the best numerical estimate you can, based on the graph of $v(t)$.

Solution: We need to find the area of the region under the curve $v(t)$ between $t = 0$ and $t = 20$, which appears to be a bit less than 3 squares. Since each square has area 150, we get an answer of a bit less than 450 meters. *Any answer in the interval* $[400, 450]$ *received credit.*

Answer: approximately 433 meters

b. [2 points] What formula would you get for $L(t)$ if you used your estimate from part **a**?

Solution: We have $L(t) = d(20) + d'(20)(t - 20)$. Since $d'(t) = v(t)$ and $v(20) = 15$ from the graph, using our approximation $d(20) \approx 433$ from part **a.** we get

$$L(t) = 433 + 15(t - 20).$$

Answer: $L(t) \approx 433 + 15(t - 20)$

c. [3 points] In each row below, circle the **one** true statement.

Solution: Note that the graph of $d(t)$ is increasing and concave down.

(i) *circle one:*	$L'(20) < d'(20)$	$L'(20) > d'(20)$	$\boxed{L'(20) = d'(20)}$
(ii) *circle one:*	$\boxed{L'(15) < d'(15)}$	$L'(15) > d'(15)$	$L'(15) = d'(15)$
(iii) *circle one:*	$L(15) < d(15)$	$\boxed{L(15) > d(15)}$	$L(15) = d(15)$

d. [2 points] Suppose that at $t = 0$, the car passes a truck on the highway. The truck is traveling in the same direction as the car, and is traveling at a constant speed of s meters per second. Eventually, the truck catches up with the coasting car and passes it at time $t = c$. How fast was the truck traveling?

Give an *exact answer* in terms of c and the function $v(t)$. Your answer may involve derivatives or integrals of $v(t)$, but it should **not** involve the function $d(t)$.

Solution: Between $t = 0$ and $t = c$, the truck travels a total of sc meters, while the car travels $\int_0^c v(t)\,dt$ meters. Therefore

$$sc = \int_0^c v(t)\,dt, \qquad \text{so} \qquad s = \frac{1}{c}\int_0^c v(t)\,dt.$$

Answer: $s = \frac{1}{c}\int_0^c v(t)\,dt$ meters per second