

3. [5 points] Let

$$j(x) = \ln(x^2 + 1).$$

Use the limit definition of the derivative to write an explicit expression for $j'(7)$. *Your answer should not involve the letter j . Do not attempt to evaluate or simplify the limit.* Write your final answer in the answer box provided below.

Answer: $j'(7) =$ $\lim_{h \rightarrow 0} \frac{\ln((7+h)^2 + 1) - \ln(7^2 + 1)}{h}$

4. [4 points] Consider the family of circles defined by the equations

$$(x - a)^2 + y^2 = r^2,$$

where a and r are parameters with $r > 0$. Jessie draws a circle from this family that passes through $(0, 10)$ with a slope of $-\frac{1}{2}$. Find the exact value of both parameters a and r for Jessie's circle. *Show your work.*

Solution: Implicitly differentiating both sides of the given equation with respect to x (and remembering to treat a and r as constants), we get

$$2(x - a) + 2y \frac{dy}{dx} = 0.$$

Plugging in $x = 0$, $y = 10$, and $\frac{dy}{dx} = -\frac{1}{2}$, we find

$$2(0 - a) + 2(10)(-1/2) = 0, \quad \text{or} \quad -2a - 10 = 0,$$

so $a = -5$.

Now we can plug $x = 0$, $y = 10$, and $a = -5$ into the original equation and solve for r to get

$$(0 - (-5))^2 + 10^2 = r^2, \quad \text{or} \quad 25 + 100 = r^2,$$

so $r = \sqrt{125}$.

Answer: $a =$ -5 and $r =$ $\sqrt{125}$