MATH 116 — FIRST MIDTERM EXAM

Fall 2004

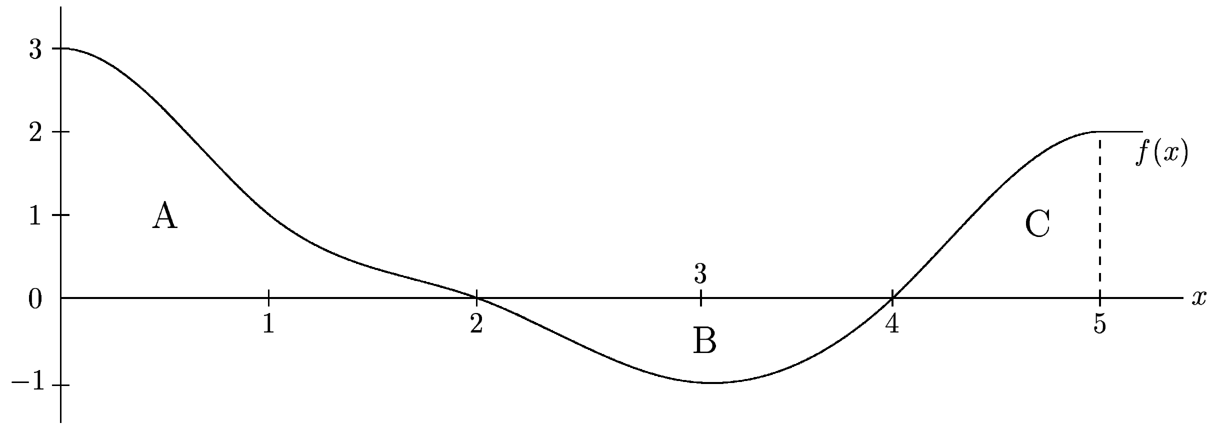
NAME: ____________________________ ID NUMBER: ____________________________

INSTRUCTOR: ____________________ SECTION NO: ____________________________

1. Do not open this exam until you are told to begin.
2. This exam has 9 pages including this cover. There are 8 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed 2 sides of a 3 by 5 notecard.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to make clear how you arrived at your solution.
8. Please turn **off** all cell phones and devices whose sounds might disturb your classmates. Please remove **all** headphones.

PROBLEM	POINTS	SCORE
1	15	
2	8	
3	8	
4	10	
5	12	
6	15	
7	18	
8	14	
TOTAL	100	

1. (15 pts.) For $0 \leq x \leq 5$, let $g(x) = \int_0^x f(t)\, dt$, where f is the function whose graph is shown in the figure. The areas of the regions bounded by the graph of f and the x-axis, and labeled A, B, C are equal to 2.5, 1, and 1, respectively.



(a) Find, as accurately as you can...

(i) the values of:

$g(2) =$ ______, $g(4) =$ ______, $g'(5) =$ ______.

(ii) the interval(s) on which g is decreasing.

(iii) the interval(s) on which g is concave up.

(iv) the value(s) of x and $g(x)$ for the value(s) of $0 \leq x \leq 5$ where $g(x)$ is largest.

(b) On the above figure, sketch as accurately as you can the graph of g. Make sure...

- that your graph is consistent with your answers to parts (a)-(d);
- to label any points on the graph where you know the coordinates of the point $(x, g(x))$.

2. (8 pts.) Circle **T** if the phrase completing the sentence is true, or **F** if it is false. You need not explain your answer.

The function $f(x) = \sin(x^2)$

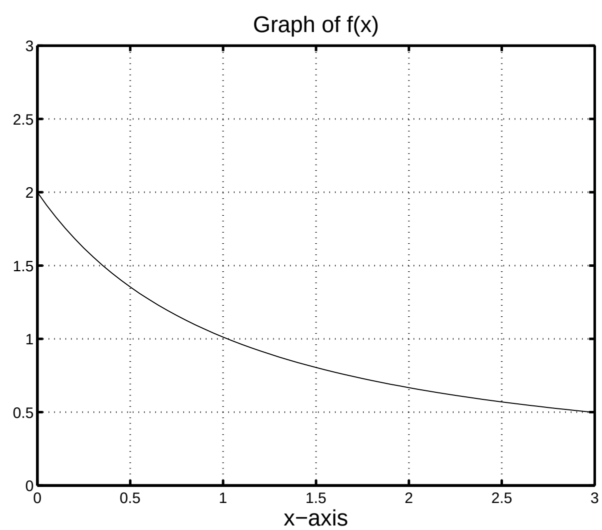
(i) *does not have an antiderivative.* **T** **F**

(ii) *has* $-\dfrac{\cos(x^2)}{2x}$ *as an antiderivative.* **T** **F**

(iii) *has exactly one antiderivative.* **T** **F**

(iv) *has antiderivative* $2x\cos(x^2)$. **T** **F**

3. (8 points) For the function whose graph is shown in the figure, the numerical values of the five numbers given by the exact value of the integral (correct to the indicated number of places), the Left Hand Sum, the Right Hand Sum, the Trapezoid Rule, and the Midpoint Rule, with $n = 60$ are listed below. Match them with the corresponding approximation, labeling each number with either EXACT, LHS, RHS, TRAP, or MID. (No explanation is required.)



2.7560 ____________

2.7930 ____________

2.7932 ____________

2.7936 ____________

2.8311 ____________

4. (10 points) Give the definition of the improper integral $\int_1^\infty \frac{1}{x^{3/2}}\,dx$. Then use your definition to evaluate the integral if it converges, or else show it diverges.

5. (12 points). Evaluate the integrals, given that $f(x)$ is a continuous function for $0 \le x \le 6$ with the following
properties:

$$f(0) = 2, \quad f(2) = 3, \quad f(4) = -1, \quad f(6) = 5; \qquad f'(0) = 1, \quad f'(2) = 4;$$

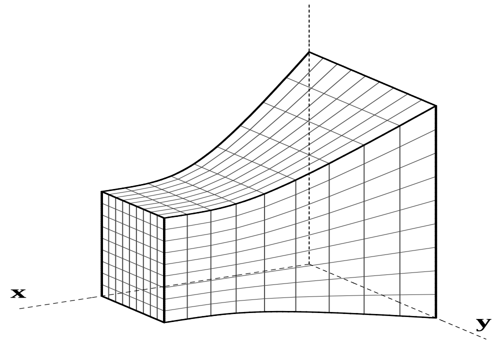
$$\int_0^2 f(x)\,dx = 3, \quad \int_2^4 f(x)\,dx = 1, \quad \int_4^6 f(x)\,dx = 6.$$

(a) $\int_0^2 x f'(x)\,dx =$ ______________.

(b) $\int_2^4 f'(x)\big(2 + 3f(x)\big)\,dx =$ ______________.

(c) $\int_0^2 f(3x)\,dx =$ ______________.

6. (15 pts.) Consider the region in the x, y plane bounded by the graph of $y = \left(1 + (2-x)^4\right)^{1/4}$, the x-axis, and the lines $x = 0$ and $x = 2$.

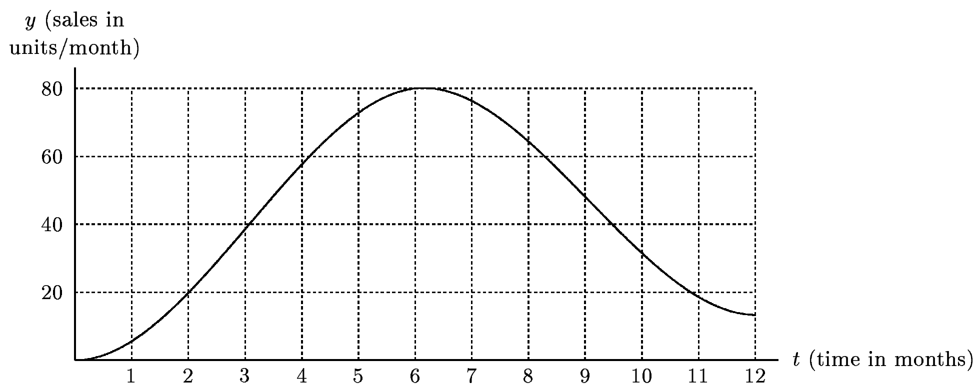


(a) Write an integral giving the value of the volume of the solid whose base is the given region and whose cross-sections perpendicular to the x-axis are squares. (A three-dimensional view of the solid is presented in the figure.)

(b) Explain how Riemann sum approximations of your integral are related to approximations of the volume of the solid.

(c) Find, as accurately as you can, the value of the volume of the solid described in part (a). Explain how you computed your answer.

7. (18 points) Aberister is a small company which sells clothing items for youngsters in North America. When the new designs are released on January 1, 2005, the company projects sales $y(t)$ of shirts (in units of 1,000 items per month) for the year 2005 to be as shown in the figure. As the figure shows, sales are expected to increase until the summer when most "target" customers will have made their purchases, and then decline for the remainder of the year.



(a) Write an expression for $A(t)$, the projected monthly average number of units which will have been sold in the first t months of the year.

$A(t) =$

(b) Why is the value of $A(t)$ always less than 80, for any value of $0 < t \leq 12$?

(c) Approximate the value of $A(t)$ at $t = 3$. Explain how you arrived at your approximation.

ANSWER: $A(3) =$ ________________

Problem continued on next page.

Problem continued from previous page.

Assuming the price of the shirts remains constant throughout the year, Aberister will maximize its profit on the 2005 designs by launching its next collection, the holiday seasons designs, when $A(t)$ is maximum.

(d) Suppose that at time t_{max} the value of A is maximum. What is the relationship between the values of y and A at time t_{max}? Explain.

(e) Sketch a graph of the function $A(t)$ on the above figure (previous page). Be sure to show where $A(t)$ is increasing and decreasing. Use your graph to estimate the time of the year when Aberister should launch its new collection.

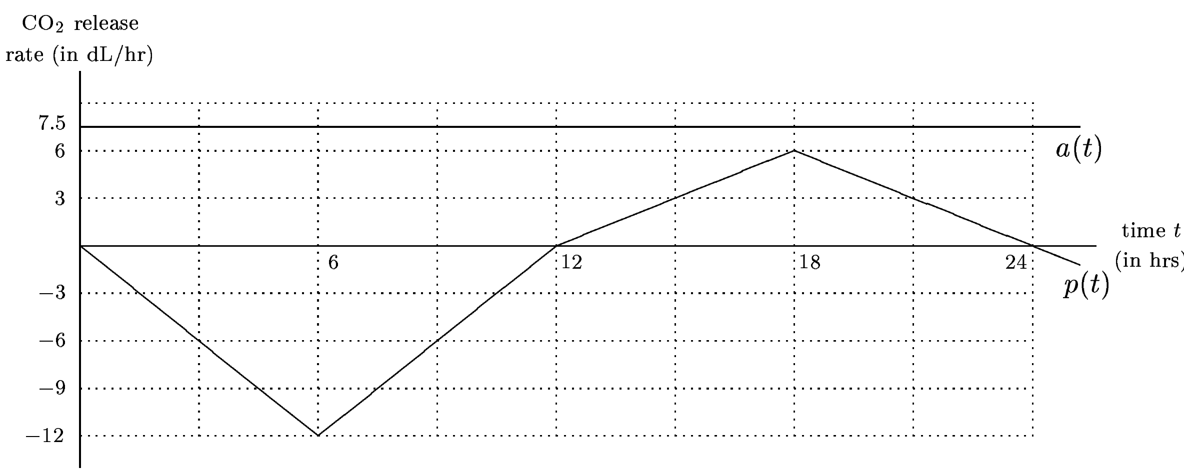
8. (14 points) A team of biologists seeking to develop alternate solutions to the use of pesticides proposes the following experiment. A plant infested by a large colony of aphids (small insects), is placed in a container originally saturated in dioxygen (O_2). They hope that the parasites can be suffocated by the CO_2 produced by their natural activities (e.g. breathing) and those of the plant (e.g. photosynthesis). Make the simplifying assumptions, which are approximations, that:

(i) The container is sealed; no molecules enter or leave.

(ii) The sum of the volumes of O_2 and CO_2 in the container is constant and equal to 117 deciliters (dL); i.e. 11.7 liters.

(iii) At the start of the experiment ($t = 0$), there is no CO_2 in the container.

The light in the room is adjusted so as to mimick a "perfect" 24-hour period. The parasites produce CO_2 at a constant rate of 7.5 dL per hour. When the light is turned on, the plant absorbs CO_2 and releases O_2. When the light is turned off, it does the opposite; i.e. uses O_2 and produces CO_2.
The rates of release in the container of CO_2 (in dL per hour) for the parasites, $a(t)$, and the plant, $p(t)$, are plotted below. Note that negative rates correspond to absorption.



(a) By approximately how much does the volume $V(t)$ of CO_2 in the container change in the small time interval between t and $t + \Delta t$? Express your answer in terms of Δt, and of the rates $p(t)$ and $a(t)$.

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(b) Write an integral that gives the volume $V(12)$ of CO_2 in the container after 12 hours. Explain why your integral gives the value of $V(12)$. You will probably want to use the answer from **(a)** in your explanation.

(c) What is the value of $V(12)$? Explain how you obtained your answer.

ANSWER: $V(12) =$ __________________.

(d) When the container becomes saturated in CO_2, i.e. no O_2 remains, the parasites suffocate to death, and the experiment is stopped. Decide whether that will happen within the 24-hour period; and, if you think it will, estimate this time.