Math 116 — First Exam

October 11, 2006

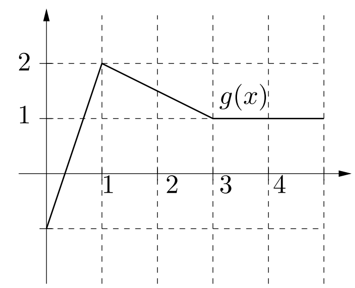
Name: ______________________________

Instructor: ______________________________ Date: ____________

1. **Do not open this exam until you are told to do so.**
2. This exam has 10 pages including this cover. There are 9 problems. Note that the problems are not of equal difficulty, and it may be to your advantage to skip over and come back to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full keyboard). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones and pagers**, and remove all headphones.

Problem	Points	Score
1	12	
2	12	
3	12	
4	8	
5	8	
6	10	
7	16	
8	12	
9	10	
Total	100	

1. [12 points] For all of parts (a)–(d), let $f(x) = 2x - 4$ and let $g(x)$ be given in the graph to the right.



(a) [3 points of 12] Find $\int_1^5 g'(x)\,dx$.

(b) [3 points of 12] Find $\int_0^5 g(x)\,dx$.

(c) [3 points of 12] Find $\int_2^{4.5} g(f(x))\,dx$.

(d) [3 points of 12] Find $\int_0^5 f(x) \cdot g'(x)\,dx$.

2. [12 points] While working on their team homework, Alex and Chris find that they have evaluated the same integral—but that they each used a different method, and got different answers! Alex found

$$\int (2x-1)(3+x)^4\,dx = (2x-1)\left(\frac{1}{5}(3+x)^5\right) - \frac{1}{15}(3+x)^6 + C.$$

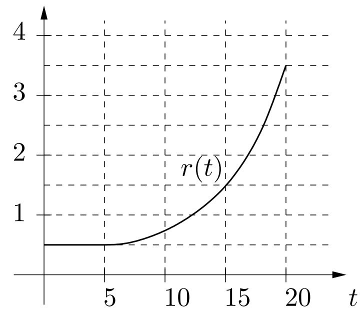
while Chris had

$$\int (2x-1)(3+x)^4\,dx = \frac{1}{3}(3+x)^6 - \frac{7}{5}(3+x)^5 + C,$$

(a) [6 of 12 points] Considering the form of the solution that Alex found, what method is it most likely that Alex used? Use this method and verify that you obtain the same solution.

(b) [6 of 12 points] Considering the form of the solution that Chris found, what method is it most likely that Chris used? Use this method and verify that you obtain the same solution.

3. [12 points] Having completed their team homework, Alex and Chris are making chocolate chip cookies to celebrate. The rate at which they make their cookies, $r(t)$, is given in cookies/minute in the figure to the right (in which t is given in minutes). After $t =$ 20 minutes they have completed their cookie making extravaganza.



(a) [3 of 12 points] Write an expression for the total number of cookies that they make in the 20 minutes they are baking. Why does your expression give the total number of cookies?

(b) [3 of 12 points] Using $\Delta t = 5$, find left- and right-Riemann sum and trapezoid estimates for the total number of cookies that they make.

(c) [3 of 12 points] How large could the error in each of your estimates in (b) be?

(d) [3 of 12 points] How would you have to change the way you found each of your estimates to reduce the possible errors noted in (c) to one quarter of their current values?

4. [8 points] Use the fact that $\int_0^\infty e^{-x}\sin(x)\,dx = \frac{1}{2}$ to find $\int_0^\infty e^{-x}\cos(x)\,dx$.

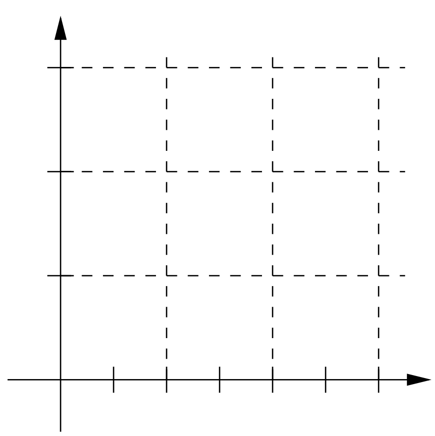
5. [8 points] Let $F(x) = \int_0^{x^2(x-1)} g(t)\,dt$, where $g(t)$ is always positive. For what values of x is $F(x)$ increasing? For what values is it decreasing?

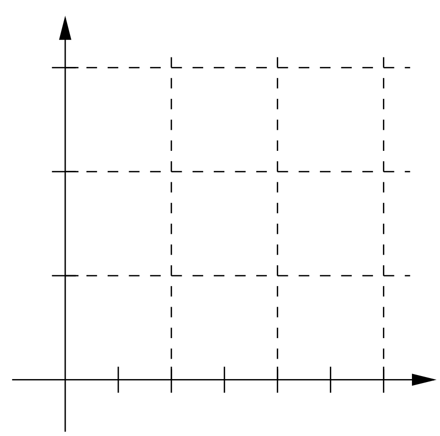
6. [10 points] While eating cookies, Alex notes that the velocity of a student passing by is given, in meters/second, by the data shown below.

t (seconds)	0	1	2	3	4	5	6
$v(t)$ (m/s)	0	0.5	1.5	2	2.5	2.5	3

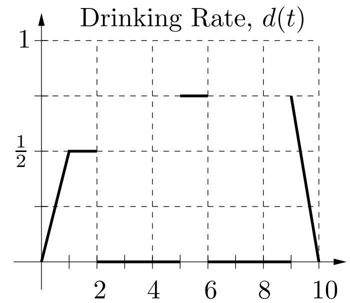
(a) [5 of 10 points] Using the midpoint rule, find as accurate an estimate as possible for the total distance the student travels in the six seconds shown in the table (use only the given data in your calculation).

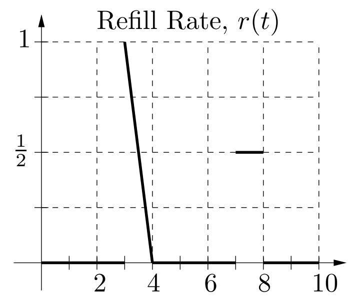
(b) [5 of 10 points] Draw two figures on the axes below, one each to illustrate the total distance you are estimating and the estimate you found. Be sure it is clear how your figures illustrate the indicated quantities.



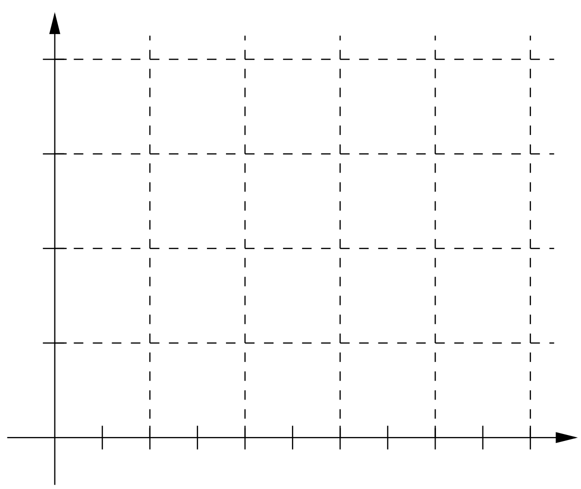


7. [16 points] As they eat their cookies, Alex and Chris are drinking milk. They each have a glass from which they are drinking, and the jug of milk is conveniently located in the middle of the table. They both start with a full glass of milk (one pint). As they are drinking, Chris sneakily refills the glass that Alex is drinking from while Alex is distracted by the student that is passing by. The rate at which Alex is drinking the milk, $d(t)$, is shown in the top figure to the right, while the rate at which Chris refills the glass, $r(t)$, is shown in the bottom figure. Both figures give the rates in pints per minute, and the time in minutes.





(a) [8 of 16 points] Let $V(t)$ be the amount (volume) of milk in the glass that Alex is drinking from, as a function of time. Carefully sketch $V(t)$ on the axes provided. Be sure to label your axes and that your sketch has accurate vertical and horizontal scales.



(b) [2 of 16 points] Does Alex ever finish all of the milk in the glass?

7. *continued: see the previous page for any information you need about the problem.*

(c) [4 of 16 points] Give, but do not evaluate, an expression that gives the average amount of milk in Alex' glass for the time interval $0 \leq t \leq 10$. Your expression may use any of the functions $d(t)$, $r(t)$ and $V(t)$, etc., given in the problem.

(d) [2 of 16 points] If you had to evaluate your expression in (c), how would you do it? (Explain in one or two sentences.)

8. [12 points] In class, Chris' calculus professor is well known to cover material at a rate $m(t) = \frac{1}{12(t-20)^{2/3}}$ textbook sections/minute, where t is the time in minutes since the start of class.

(a) [2 of 12 points] What is the meaning of the integral $\int_0^{80} m(t)\,dt$ (include units in your explanation)?

(b) [4 of 12 points] How many sections would you estimate the professor covers in the first minute of class? In the 20th minute? Why?

(c) [6 of 12 points] Find exactly (that is, by hand) the value of $\int_0^{80} m(t)\,dt$.

9. [10 points] To improve their understanding of the material in their Calculus course, Alex and Chris have invented a set of statements about the material they have been studying. These statements are given below. For each statement, circle **true** (that is, the statement is always true), or **false** (it isn't), and give a one sentence explanation for your answer.

(a) [2 of 10 points] If a bounded continuous function $f(x)$ has the properties that $f(x) > \frac{1}{x}$ for $1 < x < 50$, $f(50) = \frac{1}{50}$, and $f(x) < \frac{1}{x}$ for $x > 50$, then $\int_1^\infty f(x)\,dx$ converges.

TRUE FALSE

(b) [2 of 10 points] If a bounded continuous function $f(x)$ has the properties that $f(x) > \frac{1}{x^2}$ for $1 < x < 50$, $f(50) = \frac{1}{2500}$, and $f(x) < \frac{1}{x^2}$ for $x > 50$, then $\int_1^\infty f(x)\,dx$ converges.

TRUE FALSE

(c) [2 of 10 points] Since the function $\frac{\sin(x)+2}{\sqrt{x}}$ is always less than $\frac{3}{\sqrt{x}}$ for $2 \le x < \infty$ and $\lim_{x\to\infty} \frac{1}{\sqrt{x}} = 0$, we know that $\int_2^\infty \frac{\sin(x)+2}{\sqrt{x}}\,dx$ converges.

TRUE FALSE

(d) [2 of 10 points] If $0 < \frac{1}{x} < g(x) < \frac{1}{x^2}$ for $0 < x < 1$, then the area between $g(x)$ and the x-axis for $0 < x < 1$ is guaranteed to be finite.

TRUE FALSE

(e) [2 of 10 points] Let $f(x) = \frac{1}{(x-1)^2}$. Then if $F(x) = \int_0^x f(t)\,dt$, we know that $F(0) = 0$ and that $F(2) = \int_0^2 \frac{1}{(t-1)^2}\,dt = -\frac{1}{t-1}\Big|_0^2 = -1 - 1 = -2$.

TRUE FALSE