# Math 116 — Second Exam

November 15, 2006

Name: _____

Instructor: _____ Section: _____

1. **Do not open this exam until you are told to do so.**
2. This exam has 9 pages including this cover. There are 8 problems. Note that the problems are not of equal difficulty, and it may be to your advantage to skip over and come back to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full keyboard). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones and pagers**, and remove all headphones.

| Problem | Points | Score |
|---|---|---|
| 1 | 12 | |
| 2 | 10 | |
| 3 | 14 | |
| 4 | 12 | |
| 5 | 8 | |
| 6 | 16 | |
| 7 | 12 | |
| 8 | 16 | |
| Total | 100 | |

**1.** [12 points] While at home for Thanksgiving, Alex finds a forgotten can of corn that has been sitting on the shelf for a number of years. The contents have started to settle towards the bottom of the can, and the density of corn inside the can is therefore a function, $\delta(h)$, of the height $h$ (measured in cm) from the bottom of the can. $\delta$ is measured in g/cm$^3$. The can has a radius of 4 cm, and a height of 12 cm.

**(a)** [3 points of 12] Write an expression that approximates the mass of corn in the cylindrical cross-section from height $h$ to height $h + \Delta h$.

**(b)** [3 points of 12] Write a definite integral that gives the total mass of corn in the can.

**(c)** [3 points of 12] If $\delta(h) = 4e^{-0.03h}$, what is the total mass of corn inside the can?

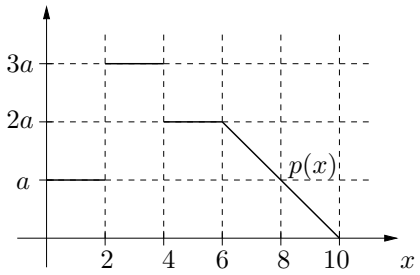
**(d)** [3 points of 12] Write, but do not evaluate, an expression for the can's center of mass in the $h$ direction. Would you expect the center of mass to be in the top or bottom half of the can? Do not solve for the center of mass, but in one sentence, justify your answer.

**2.** [10 points] Alex and Chris decide to invest money in a savings account to prepare for their expenses after they land a posh mathematical consulting job following their success in calculus. They deposit \$100 on the first of each month into an account that pays 0.4167% interest at the end of each month (an annual yield of about 5%). Let $B_n$ be the amount in their account immediately after their $n$th deposit.

**(a)** [5 points of 10] $B_n$ is a sequence. Give the first four terms in this sequence.

**(b)** [5 points of 10] Write a general, closed-form, formula for $B_n$ (your expression should involve none of the symbols $\Sigma$, $\cdots$, or $\int$).

**3.** [14 points] For the graduating class of 2010 from a major university (its name concealed so as to protect its identity), the probability density function, $p(x)$, for the number of job offers, $x$, obtained by a graduate is shown in the figure to the right. The value $a$ appearing in the values on the $y$-axis of this figure is a constant.




**(a)** [3 points of 14] What is the value of $a$?

**(b)** [3 points of 14] What is the probability that a graduate will get at least 4 but no more than 8 job offers?

**(c)** [4 points of 14] Write, but do not evaluate, an expression giving the mean number of job offers obtained by a graduate. Explain in one sentence how you would evaluate your expression.

**(d)** [4 points of 14] Write an expression that gives the median number of job offers obtained by a graduate. Use your expression to find the median.

**4.** [12 points] The following three parts of this problem have to do with the convergence of series.

**(a)** [4 points of 12] For $\Sigma \frac{5}{1+n+e^n}$:

i. [2 points of 4] What is a good test to determine the convergence of this series? Explain, in 1–2 sentences only, why this is.

ii. [2 points of 4] Determine if this series converges, diverges, or if we can't tell.

**(b)** [4 points of 12] For $\Sigma \frac{n}{n^2+5}$:

i. [2 points of 4] What is a good test to determine the convergence of this series? Explain, in 1–2 sentences only, why this is.

ii. [2 points of 4] Determine if this series converges, diverges, or if we can't tell.

*this problem is continued on the following page. . .*

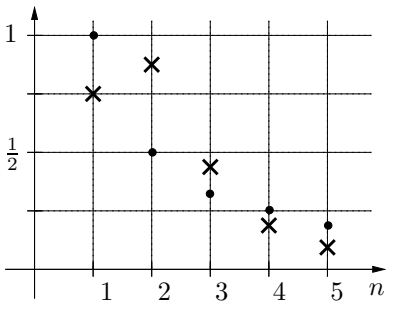
*. . . problem continued from the previous page.*

**(c)** [4 points of 12] For $\Sigma \frac{n}{2n^3-1}$:

i. [2 points of 4] What is a good test to determine the convergence of this series? Explain, in 1–2 sentences only, why this is.

ii. [2 points of 4] Determine if this series converges, diverges, or if we can't tell.

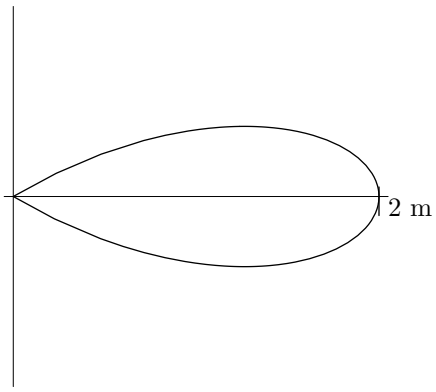
**5.** [8 points] Let $a_n$ and $b_n$ be the two sequences shown in the figure to the right. The sequence $a_n = \frac{1}{n}$ is shown with solid dots ($\bullet$) and the sequence $b_n$ is shown with crosses ($\times$). For $5 \leq n < \infty$, $0 < b_n < a_n$.




**(a)** [4 points of 8] Does the sequence $b_n$ converge, diverge, or can we not tell? Explain in one or two sentences. If it converges, indicate the value to which it converges.

**(b)** [4 points of 8] Does the series $\Sigma\, b_n$ converge, diverge, or can we not tell? Explain in one or two sentences. If it converges, indicate the value to which it converges.

**6.** [16 points] Chris has decided to take flying lessons, and notices that the cross-section of the airplane wing is given approximately by the figure to the right. The front-to-back length of the wing, as shown in the figure, is 2 m. The end-to-end length of the wing is 15 m (that is, its length along an axis coming out of this page is 15 m), and its ends are flat.




**(a)** [3 points of 16] If this cross-section is described by the polar equation $r = a\cos(3\theta)$, what is $a$?

**(b)** [4 points of 16] What range of values for $\theta$ generate this figure?

**(c)** [9 points of 16] Airplanes frequently have fuel tanks in their wings. If 75% of the wing's volume is available space for a fuel tank, what volume of fuel could be stored in this wing?

**7.** [12 points] A mysterious three-dimensional abstract sculpture has appeared on the major university's central campus. Alex, being a particularly astute calculus student, notes that the volume is given by $V = \int_1^2 (e^{-x} + 1)^2 \, dx$, where $x$ is in meters.

**(a)** [4 points of 12] What does the integrand of Alex' integral tell you about the shape of the sculpture?

**(b)** [4 points of 12] Suppose that the sculpture was placed on a set of $x$-$y$ axes. Sketch the base of the sculpture, labeling all important dimensions and features.

**(c)** [4 points of 12] Sketch and/or carefully explain what the shape of the sculpture is.

**8.** [16 points] Chris is standing at the edge of a swimming pool, holding a chain that is partially submerged in the water of the pool, as shown in the figure to the right. The chain is six feet long and weighs 5 lb/ft. When it is in the water, however, the buoyant force of the water makes the effective weight of the chain less—in the water, it weighs only 3 lb/ft. If the chain is initially half submerged in the pool and Chris lifts it straight up until it is entirely out of the water, how much work does Chris do?