Math 116 — Second Midterm

November 12, 2008

Name: _____

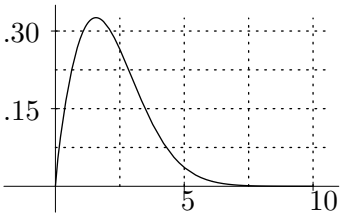
Instructor: _____ Section: _____

1. **Do not open this exam until you are told to do so.**
2. This exam has 9 pages including this cover. There are 8 problems. Note that the problems are not of equal difficulty, and it may be to your advantage to skip over and come back to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones and pagers**, and remove all headphones.

Problem	Points	Score
1	13	
2	4	
3	8	
4	16	
5	13	
6	15	
7	15	
8	16	
Total	100	

1. [13 points] A common model for the distribution of wind speeds, v, is the Rayleigh distribution. A graph of an example of the Rayleigh distribution is shown in the figure to the right, below.

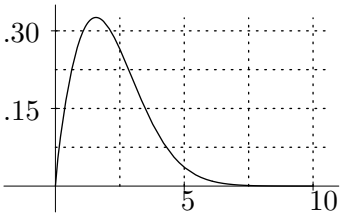
a. [2 points] Is this a *probability density function* (PDF) or *cumulative distribution function* (CDF)? Why?



b. [4 points] What is the meaning of the "5" on the x-axis of this graph? Given that the function value at 5 is 0.037, what is the meaning of the expression $(0.037)(5.01 - 4.99)$?

c. [3 points] If you identified the figure in (a) as a PDF, sketch the corresponding CDF; conversely, if you identified it as a CDF, sketch the corresponding PDF.

d. [4 points] Mark the *median* wind-speed on both the graph from (a) (reproduced below), and your graph in part (c). How did you locate the median?



2. [4 points] Let $a_n = \sum_{k=1}^{n} \frac{1}{k}$. Circle all of the statements given below that are true about the sequence a_n and *briefly* explain your answers.

-successive values of the sequence a_n increase in magnitude

-successive values of the sequence a_n decrease in magnitude

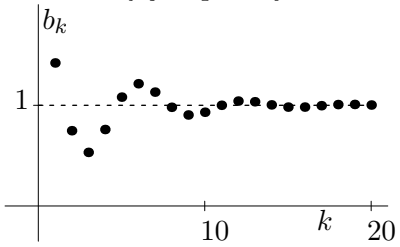
-successive values of the sequence a_n may increase or decrease in magnitude

-the sequence a_n converges

-the sequence a_n diverges

-it is not possible to determine whether sequence a_n converges or diverges

3. [8 points] Let b_k be given by the graph below (as $k \to \infty$, the behavior shown in the graph continues as is suggested by the figure). For each part of the problem below, circle all of the given statements that are true and *briefly* explain your answers.



a. [4 points] Consider the sequence b_k.

-the sequence b_k could be defined by $b_k = 1 - \frac{(-1)^k}{k}$

-the sequence b_k can only be defined recursively

-it is impossible to find a recursive definition for the sequence b_k

-the sequence b_k converges

-the sequence b_k diverges

-it is not possible to determine whether the sequence b_k converges or diverges

b. [4 points] Consider the series $\sum_{k=1}^{\infty} b_k$.

-the sequence of partial sums S_n of the series converges

-the sequence of partial sums S_n of the series diverges

-it is not possible to determine whether the sequence of partial sums S_n of the series converges or diverges

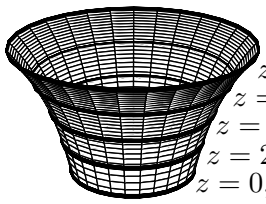
-the series $\sum_{k=1}^{\infty} b_k$ converges

-the series $\sum_{k=1}^{\infty} b_k$ diverges

-it is not possible to determine whether the series $\sum_{k=1}^{\infty} b_k$ converges or diverges

4. [16 points] An entrepreneurial University of Michigan Business Squirrel is marketing childrens' buckets with curved sides, as shown in the figure to the right, below. The figure gives the radius of the bucket, r, at different heights, z, from the bottom of the bucket. All lengths are given in inches. Suppose that a child fills one of these buckets with muddy water.

a. [4 points] If the density of the water in the bucket is $\delta(z)$ oz/in^3, write an integral that gives the mass of the water in the bucket.



b. [4 points] If $\delta(z) = (24 - z)$ oz/in^3, estimate the mass using your integral from (a).

c. [8 points] Estimate the center of mass of the bucket.

5. [13 points] Suppose that when a fire alarm is set off in East Hall, the occupants (being mathematicians) leave at precise five-minute intervals. At the end of each interval, 75% of those who were in the building at the beginning of the interval exit the building. Suppose that on a sunny Friday afternoon at 2PM a fire alarm goes off when there are 400 mathematicians in East Hall.

a. [4 points] Find the number of mathematicians that leave at the end of the first, second, and nth five-minute intervals.

b. [5 points] Let $L(n)$ be the total number of mathematicians who have left East Hall at the end of the nth five-minute interval after the alarm started. Find a closed-form expression for $L(n)$.

c. [4 points] How many mathematicians will leave the building if the alarm goes on forever? (Justify your answer mathematically.)

6. [15 points] For each of the following, assume that $\sum a_n$ and $\sum b_n$ are both convergent series, and that $a_n > b_n > 0$. For each, explain your answer in a sentence or two, or with a clear picture or counterexample.

a. [3 points] Is $\sum (b_n - a_n)$ a convergent series? Explain.

b. [3 points] Is $\sum (a_n \cdot b_n)$ a convergent series? Explain.

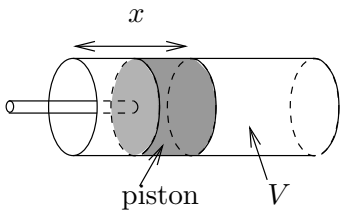
c. [3 points] Is $\sum ((-1)^n \ln(a_n + 1))$ a convergent series? Explain.

d. [3 points] Is $\sum (2\, a_n)$ a convergent series? Explain.

e. [3 points] Is $\sum ((-1)^n \sqrt{b_n}\,)$ an absolutely convergent series? Explain.

7. [15 points] Consider a piston that compresses a closed cylinder of gas, as shown in the figure to the right, below. If the volume of the gas in the cylinder is V, then the force required to move the piston and compress the gas is $F = \frac{k}{V^{1.4}}$, where k is a constant. The uncompressed length of the gas cylinder is 2 ft and its radius is $\frac{1}{4}$ ft. Let x be the distance that the piston has moved to compress the gas. (Note that the volume of a cylinder with radius r and height h is $\pi\, r^2\, h$.)

a. [5 points] Find an expression for $F(x)$, the force as a function of x. If $F(0) = 200$ lb, find k.



b. [10 points] Find the work to compress the gas from $x = 0$ to $x = \frac{3}{2}$.

8. [16 points] For each of the following series, state a convergence test that you could use to determine if the series converges or not and indicate why you chose that test. Then carefully apply the test to determine if the series converges or not. Mathematical precision is important in this problem.

a. [8 points] $\displaystyle\sum_{n=2} \frac{\sqrt{n+3}}{n^2-1}$

b. [8 points] $\displaystyle\sum \frac{(n+1)!}{2e^{3n}}$