Math 116 — First Midterm

October 14, 2009

Name: ______________________________

Instructor: ______________________ Section: ____________

1. **Do not open this exam until you are told to do so.**
2. This exam has 10 pages including this cover. There are 9 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones and pagers**, and remove all headphones.

Problem	Points	Score
1	10	
2	12	
3	12	
4	8	
5	12	
6	10	
7	11	
8	13	
9	12	
Total	100	

1. [10 points] Suppose that $f(x)$ is an odd function, $g(x)$ is an even function, and

- $\int_3^8 f(x)dx = 4$
- $\int_0^8 f(x)dx = 10$
- $\int_3^8 g(x)dx = -2$
- $\int_{-3}^3 g(x)dx = 5$

Determine each of the following quantities, if possible. If there is not enough information to determine the answer, then write "NI" in the space provided. You do not need to show your work for this page.

a. [2 points] Evaluate $\int_3^8 (f(x) - 3g(x))dx$.

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b. [2 points] Evaluate $\int_{-8}^0 g(x)dx$.

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c. [2 points] Evaluate $\int_3^8 f(x)g(x)dx$.

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d. [2 points] Evaluate $\int_{-2}^0 f(4x)dx$.

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e. [2 points] Evaluate $\int_0^3 (f(x) + 4)dx$.

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2. [12 points] A population of creatures is placed on a small preservation space. Ten creatures are initially placed on the preservation. The time it takes for a population to reach C creatures is given by

$$T(C) = \int_{10}^{C} \frac{20dx}{x(400-x)},$$

where T is measured in years after the creatures were first placed on the preservation.

a. [6 points] Find a function for $T(C)$ by analytically solving the integral given above. Be sure to show all appropriate work.

b. [2 points] How long does it take for the creatures to reach a population of 50? State your answer in a complete sentence and include units in your answer.

c. [4 points] Determine if the integral $T(400) = \int_{10}^{400} \frac{20dx}{x(400-x)}$ converges or diverges. What does your conclusion mean in terms of the creatures on the preservation?

3. [12 points] Let $g(x)$ and $h(x)$ be nonnegative functions defined for $x \geq 0$, let h be differentiable for $x \geq 0$ and suppose that

- $g(x) \leq 4$ when $x \geq 1$
- $\lim_{x\to\infty} h(x) = 0$
- $\lim_{x\to 0} \frac{h(x)}{x} = 0$
- $\int_0^\infty h(x)dx$ and $\int_0^\infty \frac{h(x)}{x^2}dx$ both converge
- $\int_0^\infty g(x)dx$ diverges

Indicate whether you think the following integrals converge, diverge, or whether there is not enough information to determine convergence. You do not need to show your work for this page.

a. [3 points] $\int_0^\infty g(x)h(x)dx$

CONVERGES DIVERGES CANNOT TELL

b. [3 points] $\int_1^\infty g(x)h(x)dx$

CONVERGES DIVERGES CANNOT TELL

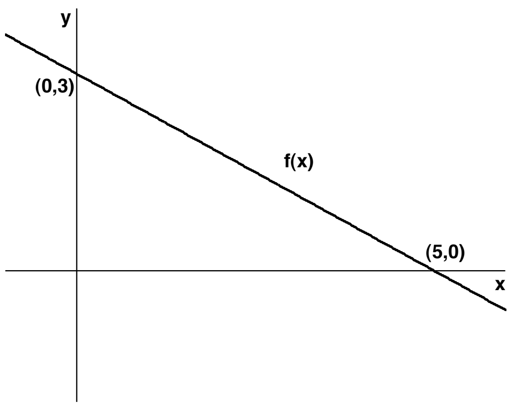
c. [3 points] $\int_0^\infty \frac{g(x)+h(x)}{2}dx$

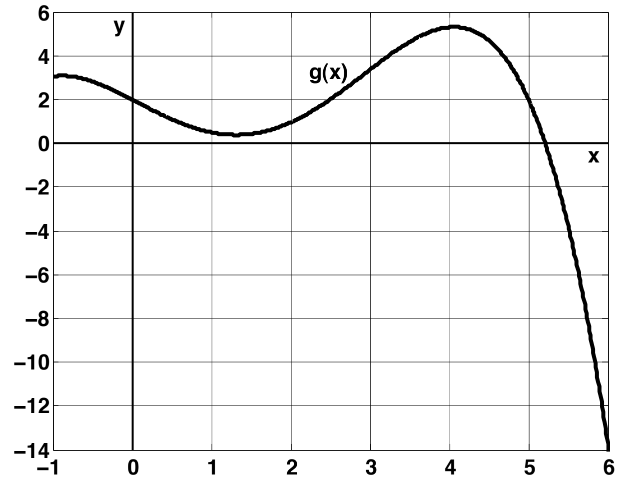
CONVERGES DIVERGES CANNOT TELL

d. [3 points] $\int_0^1 \frac{h(x)}{x}dx$

CONVERGES DIVERGES CANNOT TELL

4. [8 points] The graphs of $f(x)$ and $g(x)$ are shown below. Suppose that $f(x)$ is a linear function. Estimate $\int_0^5 f(x)g'(x)dx$. Be sure to show appropriate work to support how you derived your answer.





5. [12 points] A right isosceles triangle is a right triangle whose sides containing the right angle are of equal length. The length from the triangle's hypotenuse to its right-angle vertex (opposite of the hypotenuse) is half the length of the hypotenuse.

Consider the solid whose cross sections perpendicular to the x-axis are right isosceles triangles, where the hypotenuse of each triangular cross-section is contained in the region of the xy-plane bounded by the curves $y = \sin(x)$ and $y = -\sin(x)$ between $x = 0$ and $x = \pi$.

a. [3 points] Find the volume of the cross-sectional slice located at $x = x_i$ with thickness Δx.

b. [3 points] Write a Riemann sum that approximates the volume of the entire solid using n cross-sectional slices.

c. [6 points] Find the exact volume of the solid by using a definite integral.

6. [10 points] The distance between two points (x_1, y_1) and (x_2, y_2) is given by

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Consider the curve described by $y = \sqrt{3x^2 - 3}$, over the domain $2 \leq x \leq 4$. What is the average distance of the points on this curve to the point $(2, 0)$?

7. [11 points] A land surveyor is hired to measure the area of a plot of land to be sold. The surveyor uses two main highways as points of reference while measuring the property. Highway 116 is south of the property and runs perfectly in the east-west direction. Highway 1 is west of the property and runs perfectly in the north-south direction. The surveyor starts at Highway 1 and moves eastward for the entire four-mile width of the property as he measures the distances of the northern and southern borders of the property from Highway 116. Let $n(x)$ and $s(x)$ be the distances, in miles, of the northern border and southern borders, respectively, from Highway 116 when he is x miles east of Highway 1. The surveyor's measurements are recorded in the table below.

x	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$n(x)$	13.6	13.5	12.9	12.7	12.4	12.0	11.4	11.2	10.9
$s(x)$	7.8	8.1	8.2	8.5	8.7	8.8	9.1	10.0	10.9

a. [4 points] Estimate the area of the property using the midpoint rule with four subintervals. Be sure to show all appropriate work and don't forget to include appropriate units.

b. [4 points] Estimate the area of the property using the trapezoid rule with four subintervals. Be sure to show all appropriate work and don't forget to include appropriate units.

c. [3 points] Because he took calculus, the surveyor knows that he can determine Δx, the uniform distance at which he should make measurements in order to ensure the measured area is within a desired level of accuracy. Given that $n(x)$ is a decreasing function and $s(x)$ is an increasing function, determine the value of Δx the surveyor should use in order to measure the area within 0.5 square miles if he is using left- and right-hand Riemann sum approximations for the area.

8. [13 points] Let $C(u)$ be a function that satisfies $C'(u) = \frac{\cos(u^2)}{u}$, $C(2) = 3$, and let $S(u)$ be a function that satisfies $S'(u) = \frac{\sin(u^2)}{u}$, $S(2) = -1$.

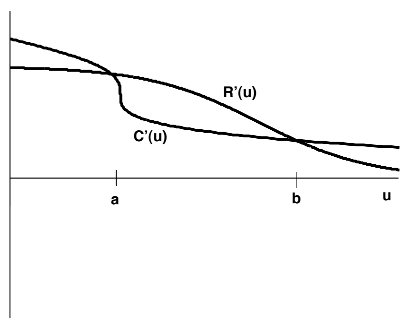
a. [4 points] Write expressions for $C(t)$ and $S(t)$ that satisfy the above conditions.

b. [5 points] A particle traces out the curve given by the parametric equations $x(t) = C(\ln(t))$, $y(t) = S(\ln(t))$ for $t \geq 10$. What is the speed of the particle at time t? You may assume that $t \geq 10$.

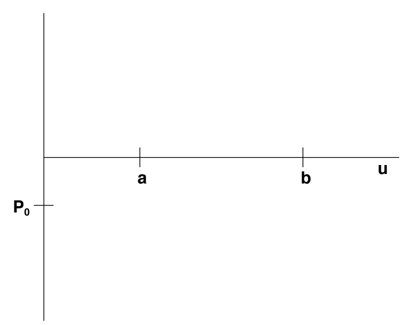
c. [4 points] For $t \geq 10$, is the curve given by the parametric equations in part (b) of finite or infinite length? Justify your answer.

9. [12 points] The primary objective of most manufacturing companies is to produce and sell the number of units that will generate the maximum profit for the company. Let $R(u)$ define the revenue income the company earns when selling u units, and let $C(u)$ define the cost of producing u units. Then the profit, P, of selling and producing u units is determined by $P(u) = R(u) - C(u)$, where profit, revenue, and cost are all measured in dollars.

a. [4 points] When trying to determine if it is beneficial to produce and sell additional goods, companies will often consider the marginal revenue, defined by $R'(u)$, and the marginal cost, defined by $C'(u)$. Below is a sketch of one company's marginal revenue and marginal cost, as a function u units. On the same axes, sketch a graph of the company's marginal profit, $P'(u)$.



b. [4 points] Using your answer to part (a), sketch a graph of $P(u)$ on the axes provided below, given the conditions that $P(0) = P_0$ and $P(b) > 0$.



c. [4 points] Given that $\int_a^b R'(u)du = \$135,000$, $\int_a^b C'(u)du = \$64,000$, and the company's profit when selling b units is $52,000, determine the company's profit when selling a units. Does the company make or lose money when selling a units?