Math 116 — Second Midterm
November 18, 2009

Name: _____

Instructor: _____   Section: _____

1. **Do not open this exam until you are told to do so.**

2. This exam has 9 pages including this cover. There are 8 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.

3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.

4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.

5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.

6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.

7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.

8. **Turn off all cell phones and pagers**, and remove all headphones.

Problem	Points	Score
1	12	
2	12	
3	12	
4	14	
5	12	
6	9	
7	16	
8	13	
Total	100	

1. [12 points] A window is being rated for underwater pressure tests. The window surface is the region bounded by the x-axis, the line $x = 4$ and the graph of $y = x/2$, where all lengths are measured in feet. (Note that the density of water is 62.4 pounds per cubic foot.)

   a. [4 points] Suppose the window is lowered under water so that the top of the window is F feet below the surface. Write an expression that estimates the force due to pressure on a thin horizontal strip of the window h_i feet below the top of the window, given that the width of the strip is Δh .

   b. [5 points] Set up a definite integral that represents the total force due to pressure on the window if the top of the window is F feet under water. Evaluate this integral in order to determine the total force due to pressure on the window. Show appropriate work to support your answer, and include units in your final answer. Your answer may be in terms of F.

   c. [3 points] How deep must the top of the window be in order for there to be a total of 700 pounds of force exerted due to water pressure on the window?

2. [12 points] On November 18, 2009, Gabrielle turned one year old. Starting today, Gabrielle's first birthday, and on every birthday up through her eighteenth birthday, Gabrielle's grandmother is going to deposit $100 in an account that earns 6.7% interest, compounded annually. On the day she turns eighteen, her grandmother will make the last deposit of $100, and then give Gabrielle all of the money in the account to help her pay for college.

    a. [3 points] What is the present value of the deposit that will be made on Gabrielle's eighteenth birthday?

    b. [3 points] Write an expression for D_n, the present value of the deposit that will be made on Gabrielle's n^{th} birthday, for $1 \leq n \leq 18$.

    c. [3 points] Write an expression for B_n, the present value of the account's balance after the nth deposit has been made.

    d. [3 points] Determine the present value of the account's balance after the final deposit is made on Gabrielle's eighteenth birthday.

3. [12 points] Indicate whether you think each of the following series converges, diverges, or whether there is not enough information to determine convergence. You do not need to show your work for this page.

    a. [3 points] Suppose $\sum_{n=1}^{\infty}(a_n + b_n)$ converges. Determine the convergence of $\sum_{n=1}^{\infty} a_n$.

        CONVERGES   DIVERGES   CANNOT TELL

    b. [3 points] Suppose $\sum_{n=1}^{\infty} a_n$ converges. Determine the convergence of $\sum_{n=1}^{\infty}(a_n + 4)$.

        CONVERGES   DIVERGES   CANNOT TELL

    c. [3 points] Suppose $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{2}$. Determine the convergence of $\sum_{n=1}^{\infty}\frac{a_n}{n}$.

        CONVERGES   DIVERGES   CANNOT TELL

    d. [3 points] Determine the convergence of $\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n}$.

   CONVERGES   DIVERGES   CANNOT TELL

4. [14 points] Using a rope, it takes 8 minutes to lift a 50-pound box of dirt from the ground to a height of 20 feet above the ground. As the box is lifted, dirt falls out of the box at a constant rate such that a total of 2 pounds of dirt has been lost when it reaches the final height of 20 feet.

a. [4 points] Let h be the height, in feet, of the box above the ground. Write an expression estimating the work done in raising the box from a height of h_i feet to a height of $h_i + \Delta h$ feet, ignoring the weight of the rope.

b. [3 points] Find the total amount of work done in raising the box of dirt from the ground to the final height of 20 feet, ignoring the weight of the rope.

c. [7 points] Suppose the rope lifting the box of dirt weighs 1.5 pounds per foot and dangles from a platform that is 30 feet above the ground. Find the total work done to lift the box of dirt to a height of 20 feet above the ground, taking into account the weight of the rope.

5. [12 points] Consider the graph of the curve $r = f(\theta) = 2 + 2\cos(\theta)$.

    a. [5 points] What is the minimal value of x taken by the points on the graph of $f(\theta)$? Show appropriate work to justify your answer.

    b. [3 points] Consider the vertical line $x = a$, where a is the value you found in part (a). Sketch the region bounded by $x = a$ and the graph of $f(\theta)$. Be sure to label your axes and graph appropriately.

    c. [4 points] Find inequalities for θ and r that describe the region bounded in part (b).

6. [9 points] Camp Summerama is a summer camp for teenagers. The camp is open for eight weeks every summer, and campers are able to attend for any length of time desired, between 0 to 8 weeks. The function $p(x)$ is the probability density function that the campers will enroll for x number of weeks. It is a piecewise function, defined by

$$p(x) = \begin{cases} \frac{c}{5}x & 0 \le x \le 5 \\ -\frac{c}{3}x + \frac{8c}{3} & 5 < x \le 8 \end{cases}$$

and shown in the graph below.
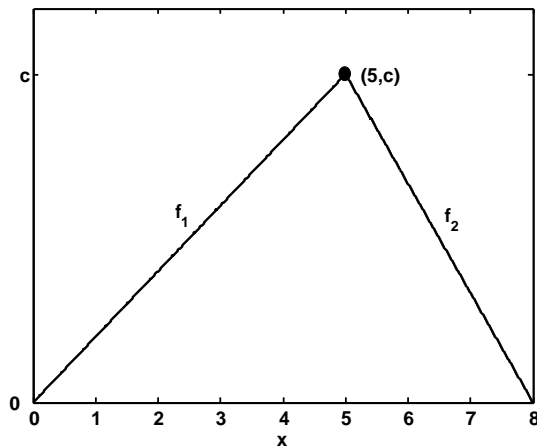
a. [2 points] What is the value of c?

b. [3 points] Evaluate $p(7)$. Interpret your answer in a complete sentence, using the context of campers and weeks spent at camp.

c. [4 points] Determine the median value for this density function. Interpret your answer in a complete sentence, using the context of campers and weeks spent at camp.
Half of the campers spend less than 4.472 weeks at camp, and half spend more than 4.472 weeks at camp.

7. [16 points] Consider the following two functions defined by power series:

$$f(x) = \sum_{n=0}^{\infty} a_n(x+1)^n \text{ and } g(x) = \sum_{n=0}^{\infty} b_n(x-2)^n,$$

where $a_n, b_n > 0$ for every n, and $a_n \to 0$ as $n \to \infty$. Both $f(x)$ and $g(x)$ converge at $x = -2$, but $f(x)$ diverges at $x = 0$ and $g(x)$ diverges at $x = 7$.

a. [4 points] What is the interval of convergence for the function $f(x)$? Justify your answer using complete sentences.

b. [4 points] Give lower and upper bounds for the radius of convergence for the function $g(x)$. Justify your answer using complete sentences.

c. [4 points] Write the series $\sum_{n=0}^{\infty} \left(b_n + (-1)^n \frac{a_n}{2^n}\right)$ in terms of f and g to determine if this series converges, diverges, or it is impossible to tell. Give a brief explanation of your answer using complete sentences.

d. [4 points] Suppose we also know that $\lim_{n \to \infty} \frac{b_{n+1}}{b_n} = \frac{1}{5}$. What is the radius of convergence of the power series $h(x) = \sum_{n=0}^{\infty} b_{2n+1} x^n$? *(Hint: You might find it helpful to use the fact that $\frac{a}{c} = \left(\frac{a}{b}\right)\left(\frac{b}{c}\right)$.)*

8. [13 points] Consider a solid metal sheet bounded by the curves $y = x^2$ and $y = ax$, for constant $a > 0$. The density of the sheet is given by $\delta(x) = 4$ grams per square centimeter.

    a. [3 points] Sketch the area of the metal sheet in the space provided below. Be sure to label your graphs and axes.

    b. [4 points] Find the exact mass of the sheet, and be sure to include appropriate units. Your answer may be in terms of a.

    c. [3 points] Find $\bar{x}$, the x-coordinate for the center of mass. Your answer may be in terms of a.

    d. [3 points] Find $\bar{y}$, the y-coordinate for the center of mass. Your answer may be in terms of a.