

# Math 116 — First Midterm

October 12, 2011

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_ Section: \_\_\_\_\_

1. **Do not open this exam until you are told to do so.**
2. This exam has 11 pages including this cover. There are 9 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a  $3'' \times 5''$  note card.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones and pagers**, and remove all headphones.

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Problem	Points	Score
1	12	
2	12	
3	9	
4	12	
5	11	
6	14	
7	13	
8	8	
9	9	
Total	100	

1. [12 points] Indicate if each of the following is true or false by circling the correct answer. No justification is required.

a. [2 points] If  $f$  and  $g$  are continuous functions over the interval  $[a, b]$ , then the average value of  $f(x)g(x)$  over that interval is the average value of  $f$  times the average value of  $g$  over that interval.

True                  False

b. [2 points] The units of  $\int f(x)dx$  are the same as the units of  $f(x)$ .

True                  False

c. [2 points] If  $f(x)$  is even and  $\int_0^2 f(x)dx = 3$ , then  $\int_{-2}^2 (f(x) - 4)dx = -10$ .

True                  False

d. [2 points] The center of mass of an object can be outside of the object.

True                  False

e. [2 points] Over the interval  $[0, 1]$ , if  $\text{LEFT}(2) = \text{RIGHT}(2)$  for a continuous function  $f(x)$ , then we know

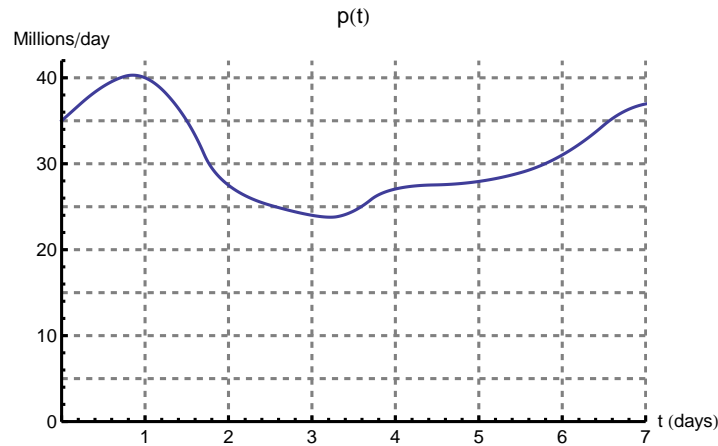
$$\text{LEFT}(2) = \int_0^1 f(x)dx = \text{RIGHT}(2).$$

True                  False

f. [2 points] Let  $f(x) > 0$  be a continuous function. Then  $F(x) = \int_0^x f(t)dt \geq 0$  for all values of  $x$ .

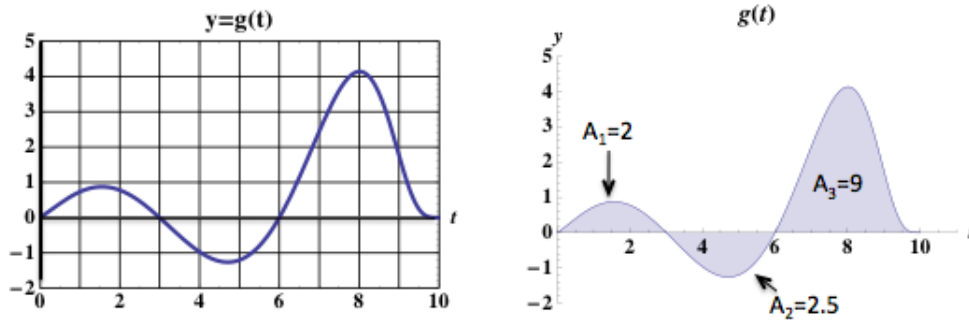
True                  False

2. [12 points] Photo sharing through social networking sites has become increasingly popular over the years. Suppose  $p(t)$  gives the rate at which photos are uploaded to Facebook's servers, over a certain one-week period, in millions of photos per day. ( $t = 0$  corresponds to the beginning of Sunday.) A graph of  $p(t)$  is given below.



- a. [2 points] Write a definite integral that gives the total number of photos uploaded to Facebook from the beginning of Sunday through the end of Monday. Include units in your answer.
- b. [8 points] Estimate the value of the definite integral in part (a) using LEFT(2), RIGHT(2), MID(2) and TRAP(2). Write each sum in terms of  $p$ .
- c. [2 points] Give a real world interpretation of the quantity  $\frac{1}{5} \int_1^6 p(t) dt$ . Include units.

3. [9 points] The graph of  $g(t)$  and the areas  $A_1$ ,  $A_2$  and  $A_3$  between its graph and the  $t$  axis are shown below.

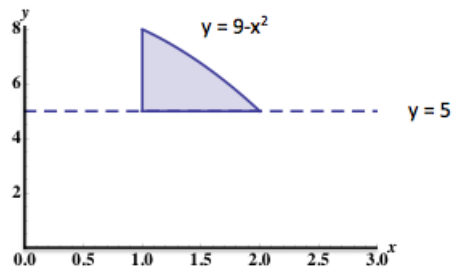


Let

$$H(x) = \int_3^{3x} g(t) dt \quad \text{and} \quad F(x) = \int_0^x g(t) dt.$$

- a. [5 points] Find  $H(1)$ ,  $H(2)$  and  $H'(3)$ .
- b. [2 points] For what values of  $5 \leq x \leq 10$  is  $F(x)$  increasing?
- c. [2 points] For what values of  $5 \leq x \leq 10$  is  $F(x)$  concave up?

4. [12 points] Consider the region in the  $xy$ -plane bounded by the curves  $y = 9 - x^2$ ,  $x = 1$ , and  $y = 5$ . This region is pictured below.



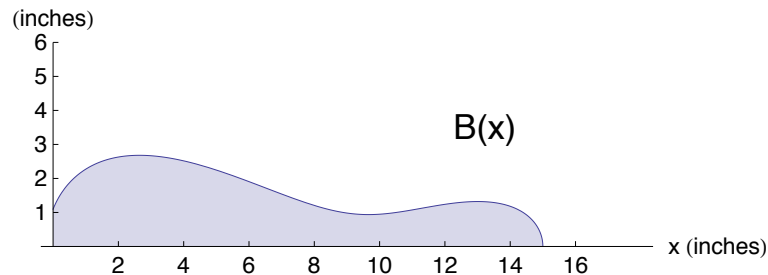
Give a definite integral that computes the quantities below. You do not need to evaluate these integrals.

- a. [3 points] The area of the region shown.
- b. [3 points] The volume of the solid obtained by rotating the region about the  $y$ -axis.
- c. [3 points] The volume of the solid obtained by rotating the region about the  $x$ -axis.
- d. [3 points] The volume of the solid obtained by rotating the region about the line  $y = 5$ .

5. [11 points] During a friendly game of ten-pin bowling, your friends Walter and Smokey begin to quarrel over whether Smokey's toe slipped over the foul line. Meanwhile, you decide to pass the time by finding a mathematical model for the shape of a bowling pin. After some careful thought, you find that a fallen pin is a solid of revolution given by rotating the region under the curve

$$B(x) = \sqrt{1.2 + 5.32x - 1.485x^2 + .135x^3 - .004x^4}$$

over the interval  $[0, 15]$  about the  $x$ -axis. The region is pictured below. All measurements are in inches. A helpful stranger in the bowling alley informs you that the wood used to make the pin has density  $\delta = 17$  grams per cubic inch.



- a. [3 points] Write a definite integral that gives the mass of the bowling pin. You do not need to evaluate this integral.
- b. [6 points] What are the coordinates  $(\bar{x}, \bar{y})$  of the bowling pin's center of mass? You may use your calculator to answer this question.
- c. [2 points] Suppose the wood used to make the pin had density  $\delta = 16$  grams per cubic inch. How does this affect the position  $(\bar{x}, \bar{y})$  of the center of mass?

6. [14 points] A botanical garden has the shape of the region in the  $xy$ -plane bounded by the curve  $y = x^2$  and the  $x$ -axis, with  $0 \leq x \leq 8$ . One of the responsibilities of the gardener, is to keep the garden free of a poisonous weed. The density  $\delta$  of the weed at any point in the garden depends on the distance  $x$  from the  $y$ -axis. Values of  $\delta$  are given in kg of plants per meter square in the table below.

$x$ (meters)	0	2	4	6	8
$\delta(x)$	10	12	15	17	18

- a. [3 points] Write an integral that computes the total amount of weed in the garden. Include units.
- b. [3 points] Compute RIGHT(4) for the integral in (a). Write out all the terms in the sum. Does this sum give an overestimate or an underestimate for the total amount of weed in the garden? Justify.

- c. [2 points] Which of the following approximations to (a) are computable with the given data? Circle all that apply.

MID(1)    MID(2)    MID(3)    MID(4)

- d. [1 point] Which Riemann sum gives the best estimate for the integral in (a)? Circle one.

RIGHT(4)    LEFT(4)    TRAP(4)

- e. [5 points] The gardener built a fence around the garden. How long is the fence? Include units. You may use your calculator.



7. [13 points] Household electricity in the United States is supplied in the form of an alternating current that varies sinusoidally with a frequency of 60 cycles per second (Hz). The voltage is given by the equation

$$E(t) = 170 \sin(120\pi t),$$

where  $t$  is given in seconds and  $E$  is in volts.

- a. [7 points] Using integration by parts, find  $\int \sin^2 \theta d\theta$ . Show all work to receive full credit. (Hint:  $\sin^2 \theta + \cos^2 \theta = 1$ .)

- b. [6 points] Voltmeters read the root-mean-square (RMS) voltage, which is defined to be the square root of the average value of  $[E(t)]^2$  over one cycle. Find the exact RMS voltage of household current.

8. [8 points] Let  $f$  be a differentiable function with derivative  $f'$ . A table of values for  $f$  and  $f'$  is given below.

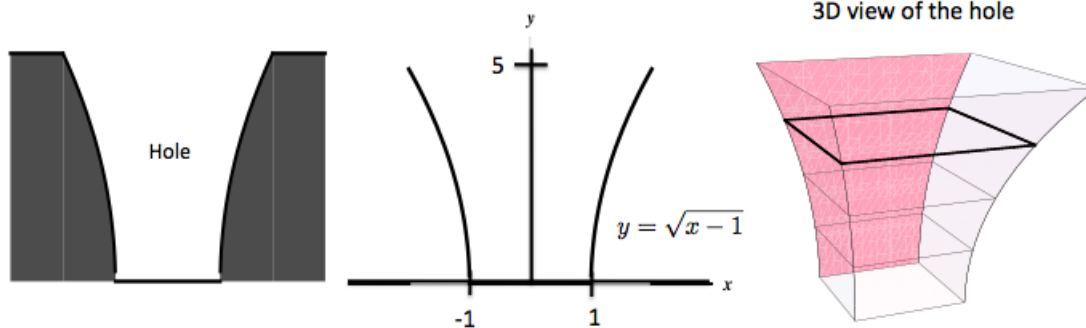
$t$	0	3	6	9
$f(t)$	1	2	7	5
$f'(t)$	1	4	-1	-2

Find the exact value of the following integrals.

a. [3 points]  $\int_0^1 f'(3t)dt.$

b. [5 points]  $\int_3^9 t f''(t)dt.$

9. [9 points] As part of an exploration assignment, a team of mining engineers dug a hole in the ground. The hole takes the shape of a solid region of known cross-section. The base region, which stands vertically, is pictured below. Cross-sections taken perpendicular to the  $y$ -axis are squares with one side lying on the  $xy$ -plane.



The variables  $x$  and  $y$  are given in meters.

- a. [6 points] Take a slice of soil of thickness  $\Delta y$  meters located at  $y$  meters above bottom of the hole. Write an expression that approximates the amount of work necessary to move that slice of soil to the top of the hole. The density of the soil is  $1600 \frac{\text{kg}}{\text{m}^3}$ . Show all work to receive full credit.
- b. [3 points] How much work does it take to dig the hole? You may use your calculator to answer this question. Include units.