Math 116 — First Midterm — October 9, 2017

EXAM SOLUTIONS

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 11 pages including this cover. There are 10 problems.
   Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Do not separate the pages of this exam. If they do become separated, write your UMID (not name) on every page and point this out to your instructor when you hand in the exam.
5. Note that the back of every page of the exam is blank, and, if needed, you may use this space for scratchwork. Clearly identify any of this work that you would like to have graded.
6. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
7. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it.
8. The use of any networked device while working on this exam is not permitted.
9. You may use any one calculator that does not have an internet or data connection except a TI-92 (or other calculator with a "qwerty" keypad). However, you must show work for any calculation which we have learned how to do in this course.
   You are also allowed two sides of a single $3'' \times 5''$ notecard.
10. For any graph or table that you use to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how you used the graph or table to find the answer.
11. Include units in your answer where that is appropriate.
12. Problems may ask for answers in *exact form*. Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is not.
13. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones, earbuds, and smartwatches. Put all of these items away.
14. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	12	
2	13	
3	5	
4	10	
5	10	

Problem	Points	Score
6	8	
7	12	
8	10	
9	8	
10	12	
Total	100	

1. [12 points] The table below gives several values of a decreasing, differentiable function G.

x	-4	-3	-2	-1	0	1	2	3	4
$G(x)$	7	5	4	2	-2	-3	-6	-8	-9

a. [4 points] Use the midpoint rule with 3 subintervals to estimate $\displaystyle\int_{-4}^{2} \big(tG(t)+4\big)\,dt$.

Carefully write out each of the terms involved in your estimate.
You do **not** have to simplify. However, no variables or function names should appear in your answer.

Solution:

$$\begin{aligned}\text{MID}(3) &= 2\cdot[(-3)G(-3)+4] + 2\cdot[(-1)G(-1)+4] + 2\cdot[(1)G(1)+4]\\ &= 2(-15+4)+2(-2+4)+2(-3+4) = -16\end{aligned}$$

So $\displaystyle\int_{-4}^{2}\big(tG(t)+4\big)\,dt \approx \text{MID}(3) = \boxed{-16}$.

In parts **b** and **c** below, calculate the <u>exact</u> numerical value of the integral.
If it is not possible to do so, write "NOT POSSIBLE". *Show each step of your work clearly.*

b. [3 points] $\displaystyle\int_{-2}^{2} 6G'(2y)\,dy$

Solution: We use w-substitution with $w = 2y$. Then $dw = 2\,dy$ so $dy = \frac{1}{2}dw$. Note that when $y=-2$ we have $w=-4$, and when $y=2$, we have $w=4$. (These values give the new limits of integration after substitution.) Substituting gives

$$\int_{-2}^{2} 6G'(2y)\,dy = \int_{-4}^{4} 6G'(w)\frac{1}{2}\,dw = 3\int_{-4}^{4} G'(w)dw = 3(G(4)-G(-4)) = 3(-9-7) = \boxed{-48}.$$

c. [5 points] $\displaystyle\int_{0}^{3} \frac{G'(x)\,G(x)}{(2G(x)-3)(G(x)+1)}\,dx$

Solution: The substitution $w = G(x)$ reduces the problem to an integral that can be computed using partial fractions. Notice that $w=-2$ when $x=0$ and $w=-8$ when $x=3$, so we have

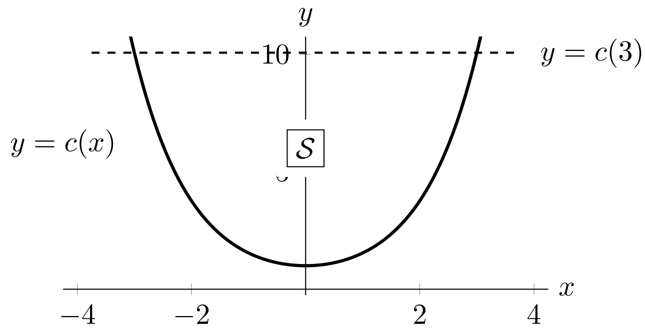
$$\begin{aligned}\int_{0}^{3} \frac{G'(x)\,G(x)}{(2G(x)-3)(G(x)+1)}\,dx &= \int_{-2}^{-8} \frac{w}{(2w-3)(w+1)}\,dw = \int_{-2}^{-8} \frac{\frac{3}{5}}{2w-3}+\frac{\frac{1}{5}}{w+1}\,dw\\ &= \left[\frac{3}{10}\ln|2w-3| + \frac{1}{5}\ln|w+1|\right]\Bigg|_{-2}^{-8}\\ &= \left(\frac{3}{10}\ln(|-19|)+\frac{1}{5}\ln(|-7|)\right) - \left(\frac{3}{10}\ln(|-7|)+\frac{1}{5}\ln(|-1|)\right)\\ &= \frac{3}{10}\ln(19) - \frac{1}{10}\ln(7) = \frac{1}{10}\ln(19^3/7) = \boxed{\frac{1}{10}\ln(6859/7)}.\end{aligned}$$

2. [13 points]

Consider the function c defined for all real numbers x by the formula

$$c(x) = \frac{e^x + e^{-x}}{2}.$$

A portion of the graph of this "catenary" function is shown as the solid curve in the graph on the right.



Let $\mathcal{S}$ be the region bounded by the graph of $y = c(x)$ and the line $y = c(3)$. This region $\mathcal{S}$ is shown in the figure above.

a. [2 points] Write, but do **not** evaluate, an expression involving one or more integrals that gives the area of $\mathcal{S}$.

Solution: Note that $c(3) = \frac{e^3+e^{-3}}{2}$ is a constant and that the graph of $y = c(3)$ intersects the graph of $y = c(x)$ at $x = \pm 3$. The area of $\mathcal{S}$ is the area between the graphs above.

$$\text{Area} = \int_{-3}^{3} \frac{e^3 + e^{-3}}{2}\,dx - \int_{-3}^{3} \frac{e^x + e^{-x}}{2}\,dx = \int_{-3}^{3} \left(\frac{e^3 + e^{-3}}{2} - \frac{e^x + e^{-x}}{2} \right) dx$$

b. [5 points] A solid is obtained by rotating the region $\mathcal{S}$ about the x-axis. Write, but do **not** evaluate, an expression involving one or more integrals that gives the volume of this solid.

Solution: Taking slices perpendicular to the x-axis, each slice is "washer"-shaped, and the volume of such a slice at x of thickness Δx is approximately $\pi\left((c(3))^2 - (c(x))^2\right)\Delta x$. The volume of the entire solid is then

$$\int_{-3}^{3} \pi\left((c(3))^2 - (c(x))^2\right) dx = \int_{-3}^{3} \pi \left[\left(\frac{e^3 + e^{-3}}{2} \right)^2 - \left(\frac{e^x + e^{-x}}{2} \right)^2 \right] dx$$

c. [3 points] Write, but do **not** evaluate, an expression involving one or more integrals that gives the arc length of the graph of $y = c(x)$ over the interval $-3 \le x \le 3$. (Your answer should **not** involve any function names.)

Solution: We first compute c' to find $c'(x) = \dfrac{e^x - e^{-x}}{2}$.

Substituting this into the formula for the arc length of the graph of a function gives

$$\text{Arc length} = \int_{-3}^{3} \sqrt{1 + (c'(x))^2}\,dx = \int_{-3}^{3} \sqrt{1 + \left(\frac{e^x - e^{-x}}{2} \right)^2}\,dx.$$

d. [3 points] Will the midpoint rule with 2000 subdivisions give an underestimate or an overestimate of the value of $\displaystyle\int_{-3}^{0} c(x)\,dx$?

Circle your answer below. Then briefly explain your reasoning in the space on the right.

Circle one:

$\boxed{\textbf{Underestimate}}$

Overestimate

Neither (They are equal)

Cannot be determined

Briefly explain your reasoning.

Solution: Note that the second derivative of c is given by $c''(x) = \frac{e^x+e^{-x}}{2}$, which is positive for all values of x (since both e^x and e^{-x} are always positive). Since c'' is always positive, the function c is always concave up. Therefore every approximation of this definite integral using the midpoint rule (including MID(2000)) will be an underestimate.[†]

[†]Note that $c'' = c$. This is a very special property of this function c.

3. [5 points] Suppose A is a differentiable function defined for all real numbers.

The function A has all of the following properties:

- A is an even function.
- $\displaystyle\int_{-2}^{2} A(x)\,dx = 5$.
- $A'(2) = 5$.
- The average value of A on the interval $[2, 4]$ is $5/2$.

Based on the properties above, circle all of the statements below that must be true.
Circle "NONE OF THESE" if none of the statements must be true.
You must circle at least one choice to receive any credit for this problem. No credit will be awarded for unclear markings. No justification is necessary.

i. $A'(-2) = 5$.

For i: In fact, $A'(-2) = -5$. (To see this, think about graph symmetry or note that A' is odd since $A'(x) = \frac{d}{dx}(A(x)) = \frac{d}{dx}(A(-x)) = -A'(-x)$.)

ii. $\displaystyle\int_{0}^{2} A(x)\,dx = 5$.

For ii: Since A is even, $\displaystyle\int_{0}^{2} A(x)\,dx = \frac{1}{2}\int_{-2}^{2} A(x)\,dx = \frac{5}{2}$.

iii. $\displaystyle\int_{2}^{4} A(x)\,dx - \int_{-2}^{-4} A(x)\,dx = 0$.

For iii: $\displaystyle\int_{2}^{4} A(x)\,dx - \int_{-2}^{-4} A(x)\,dx = \int_{2}^{4} A(x)\,dx + \int_{-4}^{-2} A(x)\,dx = 2\int_{2}^{4} A(x)\,dx$ since A is even,

iv. $\boxed{\displaystyle\int_{-2}^{2} xA'(x)\,dx = 4A(2) - 5.}$

For iv: Use integration by parts.

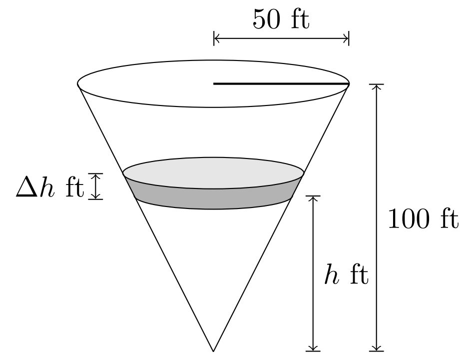
v. $\boxed{\text{The function } R \text{ defined by } R(x) = \displaystyle\int_{-x}^{x} A'(t)\,dt \text{ \underline{must} be a constant function.}}$

For v: Note that $R(x) = A(x) - A(-x)$ so R must in fact be the constant zero function.

vi. NONE OF THESE

4. [10 points]

A farming cooperative stores its alfalfa seed in a giant funnel. The funnel is in the shape of a right circular cone with height 100 feet and radius 50 feet at the top. A diagram of such a cone is shown in the figure on the right.



a. [4 points] Write an expression in terms of h that approximates the volume (in cubic feet) of a horizontal slice of the funnel of thickness Δh feet at a height of h feet above the bottom of the funnel. (Assume Δh is positive but very small.)

Solution: Using similar triangles, a horizontal cross section at height h will be a circle having radius r given by the proportion $\frac{50}{100} = \frac{r}{h}$. Hence $r = h/2$.
The approximate volume of such a slice is then

$$V_{slice} \approx \pi r^2 \Delta h = \pi (h/2)^2 \Delta h \quad \text{ft}^3$$

b. [6 points] For parts i and ii below, assume that the funnel is full of alfalfa seed. The funnel is clogged, so the alfalfa seed must be removed from above in order to clear the clog. Assume that alfalfa seed weighs 48 pounds per cubic foot.

i. Using your answer to part (a), write an expression in terms of h that approximates the work, in foot-pounds, done in moving a horizontal slice of seed of thickness Δh that is h feet above the bottom of the funnel to the top of the funnel.

Solution: The weight of such a slice is

$$\text{Weight}_{slice} = 48 \cdot (V_{slice}) \approx 48(\pi (h/2)^2 \Delta h) \quad \text{lbs}$$

This slice needs to be moved up a distance of about $100 - h$ feet. Hence the work done on such a slice is

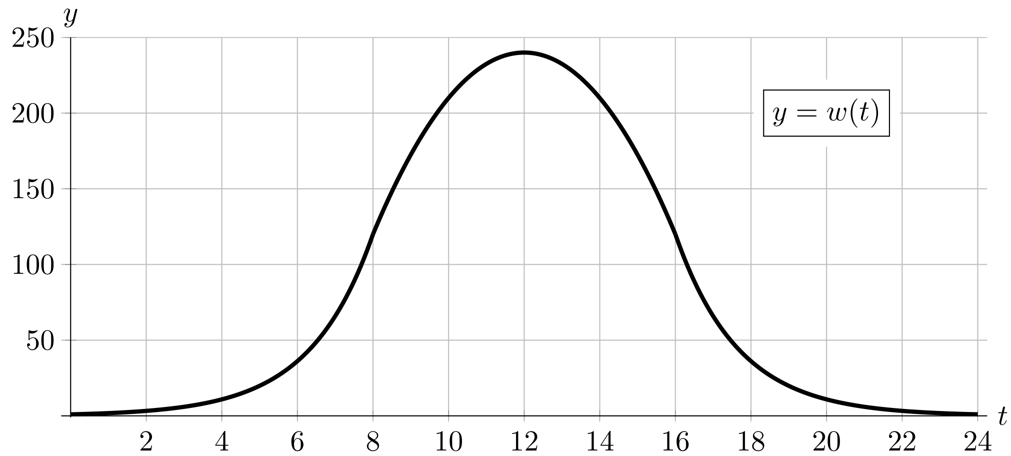
$$\text{Work}_{slice} \approx 48(100 - h)(\pi (h/2)^2 \Delta h) \quad \text{ft-lbs}$$

ii. Write, but do **not** evaluate, an expression involving one or more integrals that gives the total work, in foot-pounds, that must be done to empty the tank of seed.

Solution:

$$\text{Work}_{\text{total}} = \int_0^{100} 48(100 - h)(\pi (h/2)^2)\, dh = \int_0^{100} 12\pi h^2 (100 - h)\, dh \quad \text{ft-lbs}$$

5. [10 points] Suppose that the function $w(t)$ shown in the graph below models the power, in kilowatts, that is harvested at a particular solar panel installation in northern Norway at time t, where t is measured in hours after midnight on a typical summer day.



Consider the function W defined by

$$W(x) = \int_{2x}^{2x+4} w(t)\, dt.$$

Be sure to show your work very carefully on all parts of this problem.

a. [3 points] Estimate $W(4)$. In the context of this problem, what are the units on $W(4)$?

Solution: Note that W gives the area beneath the graph of w during a four-hour interval. In particular, $W(4) = \int_8^{12} w(t)\, dt$ is the area beneath the graph of $w(t)$ between the hours of $t = 8$ and $t = 12$. Estimating this integral (or estimating the area geometrically) gives $W(4) \approx 800$. The units on W are kilowatt·hours.

Answer: $W(4) \approx$ 800 **Units:** **kilowatt·hours**

b. [4 points] Estimate $W'(4)$. In the context of this problem, what are the units on $W'(4)$?

Solution: By the (first or second) Fundamental Theorem of Calculus together with the Chain Rule, we have

$$W'(x) = w(2x + 4) \cdot (2) - w(2x) \cdot (2).$$

Substituting $x = 4$ gives

$$W'(4) = w(12) \cdot (2) - w(8) \cdot (2) = (240) \cdot (2) - (120) \cdot (2) = 240.$$

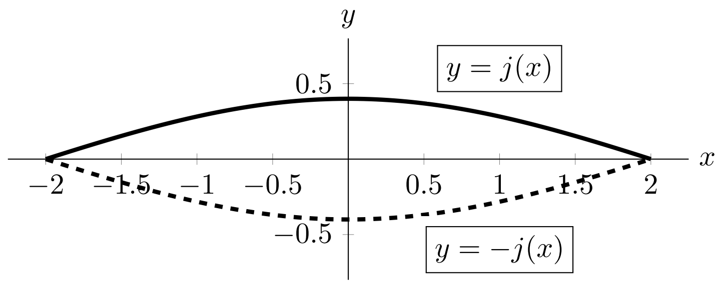
The units on W' are (kilowatts·hours)/(hours)=kilowatts.

Answer: $W'(4) \approx$ 240 **Units:** **kilowatts**

c. [3 points] Estimate the value(s) of x at which $W(x)$ attains its maximum value on the interval $0 \leq x \leq 8$. If there are no such values, explain why.

Solution: The function $W(x)$ gives the area beneath the graph of $w(t)$ during the four-hour interval between $t = 2x$ and $t = 2x + 4$. By inspecting the graph, one sees that this area is largest between the hours of $t = 10$ and $t = 14$, corresponding to $x = 5$. That is, $W(5)$ gives this maximal area, so $W(x)$ attains its maximum value at $x = 5$.

6. [8 points] A rattleback top is a toy that exhibits interesting physical properties. The toy can be modeled by a solid whose base is the region between the graphs of $j(x)$ and $-j(x)$, shown below. The cross sections perpendicular to the x-axis are semicircles.



The graph of $j(x)$ is solid, the graph of $-j(x)$ is dashed, and the units on both axes are centimeters. Both graphs are bounded between the vertical lines $x = -2$ and $x = 2$.

a. [5 points] Set up, but do **not** evaluate, an expression involving one or more integrals that gives the volume, in cubic centimeters, of the solid rattleback top. Your answer may involve the function name j.

Solution: By slicing perpendicular to the x-axis, a cross-section at horizontal coordinate x will have height $j(x) - (-j(x)) = 2j(x)$. This is the diameter of our semicircular cross-section, so the radius of the cross section is then half the diameter, or $\frac{2j(x)}{2} = j(x)$. A semicircle of radius r has area $\frac{\pi}{2}r^2$, so if each slice has thickness Δx, then the volume of such a slice can be approximated by

$$V_{slice} \approx \frac{\pi}{2}(j(x))^2\Delta x.$$

We compute the total volume by summing over all slices and taking the limit as the thickness of the slices approaches 0. The total volume is then

$$V_{total} = \int_{-2}^{2} \frac{\pi}{2}(j(x))^2\, dx \ \text{ cm}^3.$$

b. [3 points] In order to make the rattleback top spin like a top, it is made out of plastic that has a mass density given by the function $\delta(x)$ grams per cubic centimeter, where x is the x-coordinate in the diagram above. Set up, but do **not** evaluate, an expression involving one or more integrals that gives the mass, in grams, of the rattleback top. Your answer may involve the function names j and/or δ.

Solution: For small Δx, the mass of a slice as described in part **a** is approximately

$$m_{slice} \approx \frac{\pi}{2}(j(x))^2\delta(x)\Delta x.$$

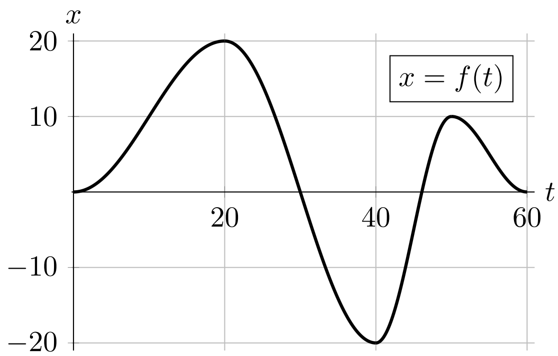
Hence the total mass is

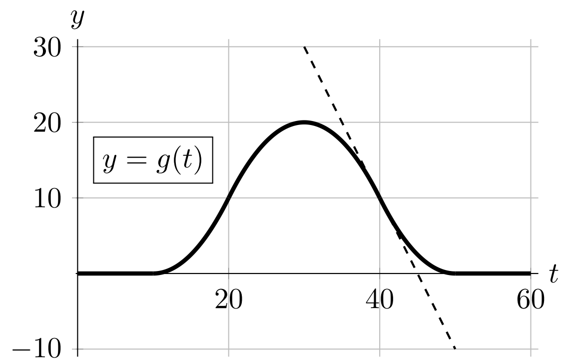
$$m_{total} = \int_{-2}^{2} \frac{\pi}{2}(j(x))^2\delta(x)\, dx \ \text{ grams}.$$

7. [12 points] Maria has a toy car that drives around her flat backyard. She describes the path of the car by typing a pair of parametric equations into a computer navigation system.

The computer controller uses x- and y-coordinates, where the units of the axes are meters, the point where Maria will be standing corresponds to the origin $(x, y) = (0, 0)$, the positive y-axis points north, and the positive x-axis points east. The car's battery will only last 60 minutes, so Maria sets the domain of each of her parametric equations to be $0 \le t \le 60$, where t is measured in minutes.

Maria enters the parametric equations $\boxed{x = f(t) \quad \text{and} \quad y = g(t)}$ where f and g are the functions shown in the graphs below.





a. [3 points] The tangent line to the graph of $y = g(t)$ at the point $t = 40$ has equation $y - 10 = -2(t - 40)$. (This is the dashed line shown in the ty-plane above.) Use this information to compute the instantaneous speed of Maria's car at time $t = 40$. Be sure to show your work clearly.

Solution: Note that $\frac{dx}{dt} = f'(t)$ and $\frac{dy}{dt} = g'(t)$. The slope of the given tangent line at $t = 40$ is the value of $g'(40)$, so $g'(40) = -2$. By examining the graph of $x = f(t)$ at $t = 40$, we also see that $f'(40) = 0$. The instantaneous speed of Maria's car at time $t = 40$ is therefore $\sqrt{(f'(40))^2 + (g'(40))^2} = \sqrt{0^2 + (-2)^2} = \boxed{\text{2 meters per minute.}}$

b. [2 points] At time $t = 0$, the car starts at Maria's location. Approximately how many meters away from Maria will the car be at time $t = 60$ (when it will run out of power)? Circle the one best estimate from among the choices below.

$\boxed{\text{0 m}}$ 150 m 300 m 450 m 600 m 750 m

c. [3 points] At which of the times listed below is the slope of Maria car's path in the xy-plane the least (most negative)? Circle the one best answer from among the choices below.

$t = 15$ $t = 20$ $\boxed{t = 28}$ $t = 32$ $t = 38$

d. [4 points] Maria's friend William programs his car to move according to the parametric equations

$$x = \int_0^t f(s)\,ds \qquad \text{and} \qquad y = \int_0^t g(s)\,ds$$

where f and g are the functions shown in the graphs above. Compute the instantaneous speed of William's car at time $t = 20$. Be sure to show your work clearly.

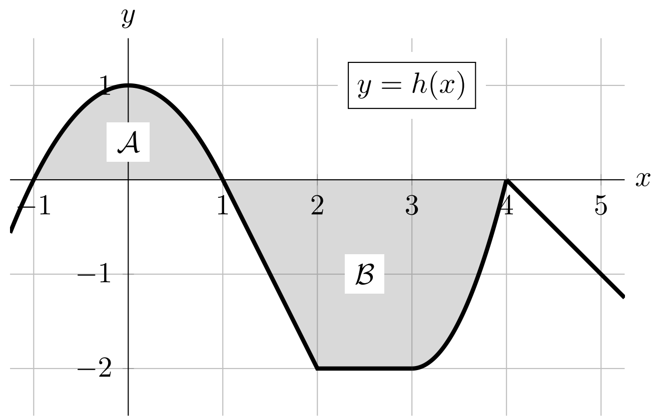
Solution: By the Second Fundamental Theorem of Calculus, we have $\frac{dx}{dt} = f(t)$ and $\frac{dy}{dt} = g(t)$. By examining the graphs, we therefore see that $\frac{dx}{dt}\Big|_{t=20} = f(20) = 20$ and $\frac{dy}{dt}\Big|_{t=20} = g(20) = 10$. Hence, the instantaneous speed of William's car at time $t = 20$ is

$$\sqrt{(20)^2 + (10)^2} = \sqrt{500} = \boxed{10\sqrt{5} \text{ meters per minute.}}$$

8. [10 points] A portion of the graph of a function h is shown below. The domain of $h(x)$ includes the interval $-1 \le x \le 5$.

Note the following:

- $h(x)$ is linear on each of the intervals $[1,2]$, $[2,3]$, and $[4,5]$.
- The portion of the graph of $y = h(x)$ for $-1 < x < 1$ is symmetric across the y-axis.
- The area of shaded region $\mathcal{A}$ is $4/3$.
- The area of shaded region $\mathcal{B}$ is $13/3$.



Throughout this problem, the function H is the antiderivative of h satisfying $H(1) = 2$.

a. [2 points] For each of the following, compute the exact value. Show your work.

i. $H(-1)$

Solution:

$$H(-1) = H(1) + \int_1^{-1} h(x)\,dx = H(1) - \int_{-1}^{1} h(x)\,dx = 2 - \frac{4}{3} = \boxed{\frac{2}{3}}.$$

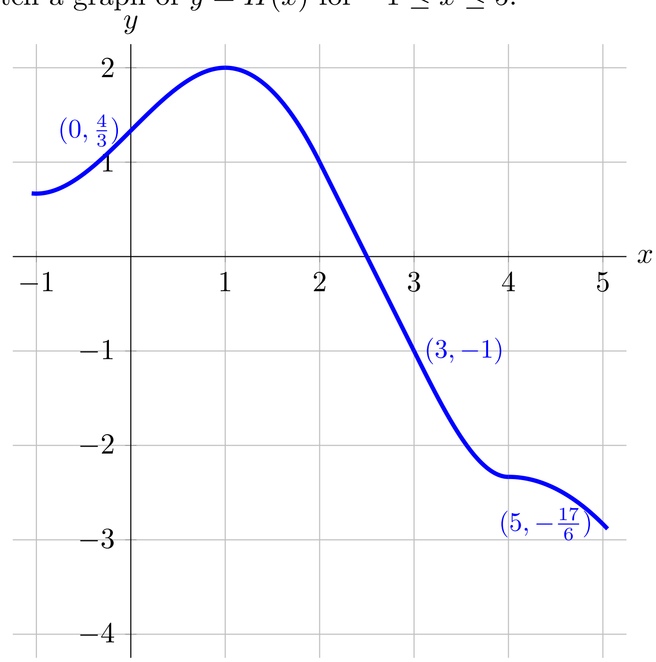
ii. $H(2)$

Solution:

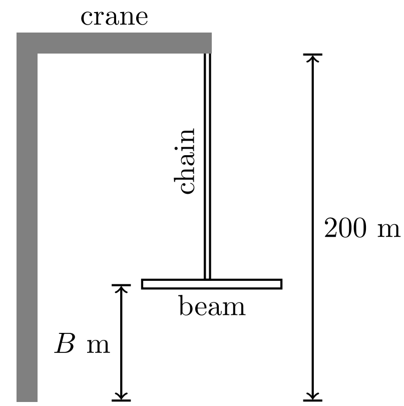
$$H(2) = H(1) + \int_1^2 h(x)\,dx = 2 + (-1) = \boxed{1}.$$

b. [8 points] Use the axes below to carefully sketch a graph of $y = H(x)$ for $-1 \le x \le 5$.

- Clearly label the coordinates of the points on your graph at $x = 0$, 3, and 5.
- Be sure that local extrema and concavity are clear.
- If there are features of this function that are difficult for you to draw, indicate these on your graph.



9. [8 points]
During the construction of a skyscraper, a 200 meter tall crane lifts a steel beam from the ground to a height of 175 meters. The steel beam has a mass of 50 kilograms. The crane has a chain that is also made of steel, and the chain has a mass of 15 kilograms per meter. The total length of the chain is 200 meters, but as the beam is lifted, the crane no longer needs to lift any of the chain that has already been "reeled in", i.e. has already reached the top of the crane.



For this problem, you may assume the acceleration due to gravity is $g = 9.8$ m/s^2 .

a. Write an expression in terms of B that gives the total mass, in kilograms, of the steel beam together with the chain that has not yet been reeled in at the moment that the steel beam is B meters above the ground.

Solution: If the steel beam is B meters above the ground, that means that a length of B meters of chain has been reeled in, so $(200-B)$ meters of chain remains. The mass of this remaining chain is (15 kilograms per meter) $\cdot$ ($200 - B$ meters) $= 3000 - 15B$ kilograms. We add to this the mass of the steel beam to find a total mass of

$$\text{Mass} = 50 + 3000 - 15B \text{ kilograms.}$$

b. Assuming ΔB is very small but positive, write an expression in terms of B that approximates the work done by the crane in lifting the steel beam up ΔB meters starting from a height of B meters above the ground. Assume that the weight of the chain being lifted is constant over this very short distance. Include units.

Solution: The force due to gravity is the weight. At the moment the steel beam is B meters above the ground, the weight of the beam together with the chain that has not yet been reeled in is

$$\text{Weight} = (\text{mass})(g) = (3050 - 15B \text{ kilograms})(9.8 \text{ m/s}^2) = 29890 - 147B \text{ Newtons.}$$

The work to lift the steel beam over the small distance ΔB is then approximately

$$(\text{Force}) \cdot \Delta B = (29890 - 147B)\Delta B \text{ Joules.}$$

c. Write, but do **not** evaluate, an expression involving one or more integrals that gives the total work that must be done by the crane in order to lift the steel beam from the ground to a height of 175 meters. Include units.

Solution: Using our answer from part **b.** above, summing over the entire path of the beam, and taking the limit as ΔB approaches 0, we find that the total work is

$$\text{Work} = \int_0^{175} g(3050 - 15B)\, dB = \int_0^{175} (29890 - 147B)\, dB \text{ Joules.}$$

10. [12 points] For each of the questions below, circle all of the available correct answers.
Circle "NONE OF THESE" if none of the available choices are correct.
You must circle at least one choice to receive any credit.
No credit will be awarded for unclear markings. No justification is necessary.

a. [4 points] Suppose a function f and both its derivative f' and second derivative f'' are defined and continuous on the entire real line $(-\infty, \infty)$. Which of the following functions must be antiderivatives of the function $t^2 f'(t)$ on $(-\infty, \infty)$?

i. $\displaystyle\int_1^t 2yf''(y)\,dy$ ii. $\boxed{\displaystyle 5 + \int_{-3}^t w^2 f'(w)\,dw}$ iii. $\displaystyle 0.25\int_0^{2t} x^2 f'(0.5x)\,dx$

iv. $\boxed{\displaystyle t^2 f(t) + \int_t^2 2xf(x)\,dx}$ v. $\displaystyle f'(1) + \int_1^4 t^2 f'(t)\,dt$ vi. NONE OF THESE

b. [4 points] Suppose that g is a function that is continuous, negative, and decreasing on the interval $[-4, 4]$ and that n is a positive integer.
Consider the definite integral $\displaystyle\int_{-4}^4 g(x)\,dx$ and the four approximations of $\displaystyle\int_{-4}^4 g(x)\,dx$ given by RIGHT(n), LEFT(n), TRAP(n), MID(n).
Which of the following could be true about the relationships between these five numbers?

i. $\boxed{\text{TRAP}(n) < \displaystyle\int_{-4}^4 g(x)\,dx}$ ii. $\boxed{\text{TRAP}(n) > \displaystyle\int_{-4}^4 g(x)\,dx}$

iii. $\boxed{\text{MID}(n) < \displaystyle\int_{-4}^4 g(x)\,dx}$ iv. $\boxed{\text{MID}(n) > \displaystyle\int_{-4}^4 g(x)\,dx}$

v. $\boxed{\text{RIGHT}(n) < \displaystyle\int_{-4}^4 g(x)\,dx < \text{LEFT}(n)}$ vi. $\boxed{\text{TRAP}(n) = \text{MID}(n)}$

vii. $\text{LEFT}(n) < \displaystyle\int_{-4}^4 g(x)\,dx < \text{RIGHT}(n)$ viii. NONE OF THESE

c. [4 points] Suppose Q is a continuous function. A circular metal plate in the xy-plane with radius 10 cm has density $Q(r)$ grams per square centimeter at a distance of r centimeters from the center of the plate. Which of the following statements must be true about this plate?

i. The total mass of the plate is $100\pi \cdot Q(10)$ grams.

ii. The total mass of the plate is $\displaystyle\int_{-10}^{10} 2\pi r \cdot Q(r)\,dr$ grams.

iii. The mass, in grams, of a very thin horizontal slice of the plate of height Δy cm located y cm above the center of the plate is approximately $Q(y)$ times the area, in cm^2, of the slice.

iv. The total mass of the plate is $\displaystyle\int_0^{10} \pi r^2 \cdot Q(r)\,dr$ grams.

v. $\boxed{\text{NONE OF THESE}}$