Math 116 — Midterm — October 5, 2020

1. **Do not write your name anywhere on this exam or your submissions.**
2. Your submissions must, however, have the correct problem number (if using blank paper) and your UMID and initials in the top corner of every page of every submission.
3. You must submit your work for each problem in Gradescope in the appropriate location.
4. Your submissions must be legible and rotated correctly.
5. This exam has 9 pages including this cover.
6. There are 9 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
7. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
8. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it.
9. You may not communicate with any other individuals, message boards, forums, etc., either in person or virtually, while you are taking the midterm.
10. You may not use a calculator, phone, computer, tablet, laptop, or any other electronic device to assist you with any question.
    You are allowed any handwritten or printed notes, provided they are printed prior to your beginning this exam.
11. For any graph or table that you use to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how you used the graph or table to find the answer.
12. Include units in your answer where that is appropriate.
13. Problems may ask for answers in *exact form*. Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is not.
14. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	14	
2	11	
3	6	
4	5	
5	15	
6	10	

Problem	Points	Score
7	15	
8	8	
9	16	
Total	100	

1. [14 points] Let $f(x)$ be a differentiable function whose derivative $f'(x)$ is also differentiable. Some values of $f(x)$ and $f'(x)$ are given in the table below:

x	0	1	2	3	4	5
$f(x)$	5	0	4	$\pi/2$	2	7
$f'(x)$	0	π	8	13	1	-6

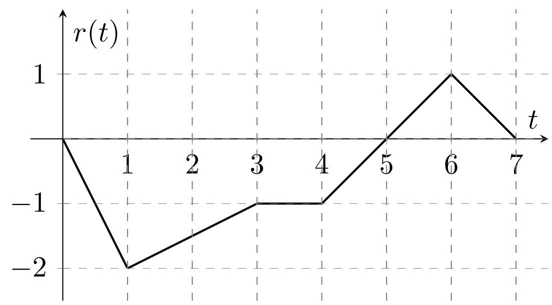
Additionally, assume $f(x)$ is positive for $x > 2$.
Compute the exact value of the following integrals. If it is not possible to do so based on the information provided, then write 'NOT POSSIBLE' and clearly indicate why it is not possible. Show all of your work.

a. [4 points] $\displaystyle\int_1^2 \frac{f'(3x-1)}{f(3x-1)}\,dx$

b. [5 points] $\displaystyle\int_0^4 (x+1)f''(x)\,dx$

c. [5 points] $\displaystyle\int_1^3 (\sin(f(x)))^3 \cos(f(x))\, f'(x)\,dx$

2. [11 points] In the game of *Vegetable Crossing*, Tina is carefully monitoring the stork market, which determines the price of a stork in dubloons, the game's currency. If t is the number of days since Tina started playing, then $r(t)$, measured in dubloons per day, gives the **rate of change** of the price of a stork in the game. A graph of $r(t)$ is shown below. Note that $r(t)$ is piecewise linear.



a. [2 points] For what value of t in $[0, 7]$ is the price of a stork growing fastest?

b. [2 points] Tina wants to buy storks when the price is as low as possible. For what value of t in $[0, 7]$ should she buy storks?

c. [3 points] What is the average value of $r(t)$ on the interval $[3, 5]$? Be sure to write down any integrals you use to obtain your answer.

d. [4 points] Let $R(t)$ be the price of a stork in dubloons at time t, and assume that $R(t)$ is continuous. The price of a stork at time $t = 3$ is 14 dubloons. Given that information, fill out the following table of values:

t	0	2	4	6
$R(t)$				

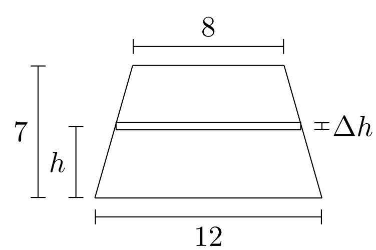
3. [6 points] Let $G(x)$ be defined by

$$G(x) = \int_{2x+3}^{5x-7} e^{t^2-1}\, dt.$$

a. [2 points] Find a value of x such that $G(x) = 0$.

b. [4 points] Find $G'(3)$.

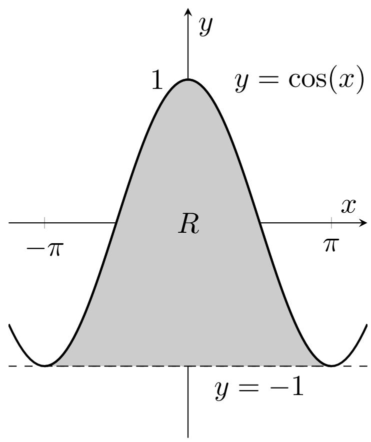
4. [5 points] A trapezoid has bases of length 12 and 8, and has height 7, as shown in the diagram below.



a. [3 points] Write an expression which approximates the area of a rectangular slice of this trapezoid with small thickness Δh at a height h from the larger base. (See the above diagram.) Your expression should not involve any integrals.

b. [2 points] Using your expression from (a) to write an integral which, when evaluated, gives the total area of the trapezoid. Do not evaluate the integral.

5. [15 points] Consider the region R in the $xy-$plane bounded between $y = \cos(x)$ and $y = -1$ for x values between $-\pi$ and π. A sketch of the region is shown below.



a. [5 points] Find an expression involving one or more integrals for the volume of the solid formed by rotating the region R around the line $x = 5$. Do not evaluate your integral(s).

b. [5 points] Find an expression involving one or more integrals for the volume of the solid formed by rotating the region R around the line $y = -3$. Do not evaluate your integral(s).

c. [5 points] Find an expression involving one or more integrals for the volume of the solid with a base in the shape of the region R, and semicircular cross sections perpendicular to the x-axis. Do not evaluate your integral(s).

6. [10 points] A thin circular plate of radius 3 m is being used to launch an electric rocket into space. The charge density, in Coulombs per m^2, on the surface of the plate a distance r meters from its center is given by a function $\delta(r) = 1 - kr$ for some constant k. Note that a Coulomb is a unit of electric charge.

a. [4 points] Write an expression involving integrals for the total charge, in Coulombs, on the surface of the circular plate. Do not evaluate the integral(s).

b. [6 points] Find the value of k if the total charge on the surface of the plate is 3π Coulombs. Be sure to show all your work including algebra and any evaluation of integrals.

7. [15 points] Consider the function

$$f(x) = \frac{1}{(x-2)^2(x-3)}$$

a. [4 points] Approximate the integral $\int_{-5}^{1} f(x)\,dx$ using MID(3). Do not decompose $f(x)$ before doing the approximation. Write out each term in any sums you make.

b. [4 points] Approximate the integral $\int_{-5}^{1} f(x)\,dx$ using TRAP(3). Do not decompose $f(x)$ before doing the approximation. Write out each term in any sums you make.

c. [7 points] Split the function $f(x)$ into partial fractions with two or more terms. Do not integrate these terms.

8. [8 points]

a. [4 points] Kesha is tasked with sourcing mahogany timber for her logging company. She finds that the cost of producing x tons of timber is given by $M(x)$, measured in thousands of dollars. She knows that $M'(x) = \ln(1 + x^2)$ and that it costs 11 thousand dollars to produce 3 tons of timber. Find an expression involving an integral for $M(x)$.

b. [4 points] Kesha finds that her department's profit for the month, in thousands of dollars, is given by

$$\int_{-5}^{5} \sin(x^5) + 2 \, dx.$$

By evaluating this integral, give a numerical figure for Kesha's department's profit. (Hint: What special property of the function $\sin(x^5)$ could be useful here?) Make sure to fully justify your answer.

9. [16 points] Archaeologists are pulling artifacts from an excavation site. As a reminder, the gravitational constant g is 9.8 m/s^2.

a. [8 points] The first artifact weighs 20 kg and lies at the bottom of the excavation site. The archaeologists are 50m above the bottom and want to use a rope to pull the artifact up to a ledge 20m from the bottom of the excavation site. If the rope has mass 2 kg per meter, how much work is done pulling the first artifact from the bottom of the excavation site to the ledge? Be sure to include units and show all your work, including calculation of any integrals.

The second artifact weighs 100 lbs when the archaeologists begin lifting it 165 feet up out of the excavation site. This artifact is crumbling, and it loses weight at a constant rate of 0.2 lbs per foot it is lifted. The cable used to lift this second artifact weighs 0.5 lbs per foot.

b. [3 points] Write a formula for $w(y)$, the weight of the second artifact in lbs, when it has been lifted y feet out of the excavation site.

c. [5 points] Write an expression involving one or more integrals for the work done pulling the second artifact out of the excavation site. Do not evaluate your integral(s).