Math 116 — First Midterm — October 4, 2022

Write your 8-digit UMID number very clearly in the box to the right.

Your Initials Only: ______ Instructor Name: ________________________ Section #: _____

1. This exam has 10 pages including this cover.
2. There are 9 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it.
5. You are allowed notes written on two sides of a $3'' \times 5''$ note card.
6. For any graph or table that you use to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how you used the graph or table to find the answer.
7. Include units in your answer where that is appropriate.
8. Problems may ask for answers in *exact form.* Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is <u>not</u>.
9. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	16	
2	15	
3	12	
4	5	
5	7	
6	10	

Problem	Points	Score
7	13	
8	12	
9	10	
Total	100	

1. [16 points] Use the table to compute the following quantities. The function $h(x)$ is **odd**, **twice differentiable**, and $h'(x) > 0$ for all x-values. Write your answer using exact form on the blank provided. If there is not enough information available to answer the question, write N.I. You need to evaluate all integrals, but you do not need to simplify your final answer.

x	0	1	2	3	4
$h(x)$	0	2	4	5	7
$h'(x)$	7	3	4	7	2

a. [4 points] $\displaystyle\int_3^4 \frac{h''(t)}{h'(t)}\,dt$

Answer: ____________________.

b. [4 points] The average value of $h'(x)$ on $[-1, 1]$

Answer: ____________________.

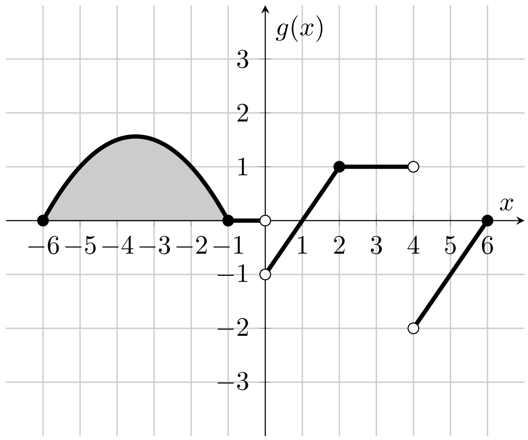
c. [4 points] $\displaystyle\int_1^4 (w+1)h''(w)\,dw$

Answer: ____________________.

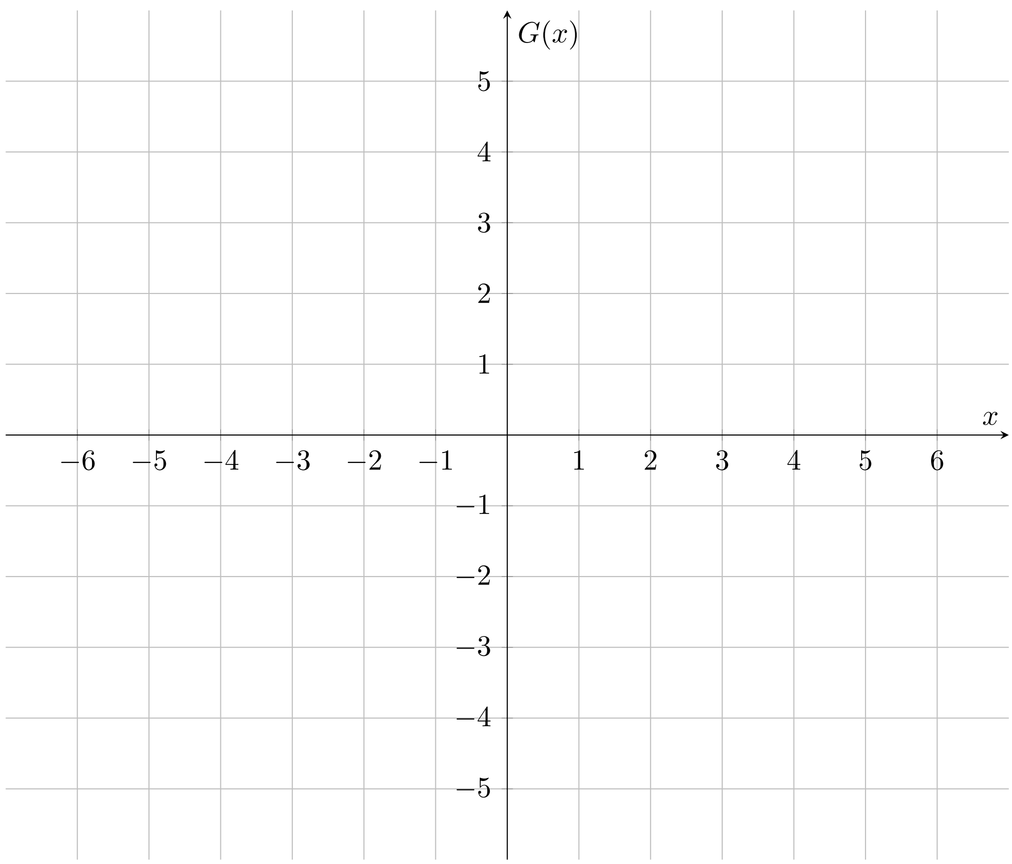
d. [4 points] $\displaystyle\int_{1/2}^2 x^{-1/2}h'(\sqrt{2x})\,dx$

Answer: ____________________.

2. [15 points] The function $g(x)$ is graphed below. The area of the shaded region is 5.5. The function $g(x)$ is piecewise linear for $x > -1$.



On the axes provided below, sketch a continuous antiderivative $G(x)$ of $g(x)$ with domain $[-6, 6]$, satisfying $G(1) = 1$. Make sure to clearly label the input and output values at $x = -6, -1, 2, 4$, and 6. Be sure to make it clear where $G(x)$ is **concave up**, **concave down**, or **linear**, and where it is **increasing**, **decreasing**, or not **differentiable**.



3. [12 points] In each part of this problem, circle all options which make the statement **always true**. There may be more than one correct answer for each part. If none of the options make the statement always true, then circle NONE OF THESE. You must circle your answers entirely to receive credit on this problem.

a. [3 points] If $f(x)$ and $g(x)$ are differentiable functions satisfying $f(0) = 0$ and $f'(x) = g(x)$, then

$g(0) = 0$.

$\pi + f(x)$ is an antiderivative of $g(x)$.

$\int_0^1 g(x)dx = f(1)$.

$2^e + g(x)$ is an antiderivative of $f(x)$.

NONE OF THESE

b. [3 points] Suppose $a < b$, and that $p(x)$ and $r(x)$ are continuous functions on $[a, b]$. If $\int_a^b p(x)\, dx < \int_a^b r(x)\, dx$ then

$p(x) < r(x)$ for all x in the interval $[a, b]$.

$p(a) < r(a)$.

$p(x) > r(x)$ for at least one value x in $[a, b]$.

$p(0) = 0$.

NONE OF THESE

c. [3 points] Suppose $k(t)$ is a continuous function with $k(t) < 0$ for all t, If $M(w) = \int_0^{-w^2} k(t)\, dt$ then

$M(w)$ is increasing for $w > 0$.

$M(0) = 0$.

$M(w)$ is increasing for $w < 0$.

$M(w)$ is decreasing for $w < 0$.

NONE OF THESE

d. [3 points] If $h(x) = x^2 + x + 1$, consider the integral $I = \int_0^{10} h(x)\, dx$. The integral I is

larger than MID(5).

larger than TRAP(5).

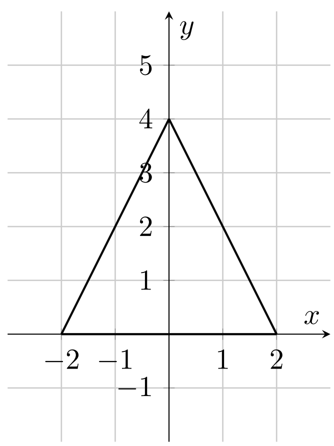
larger than LEFT(5).

larger than RIGHT(5).

NONE OF THESE

4. [5 points] Find the derivative of $f(x) = 3e^{-2x}\cos(5x)$. You do not need to simplify your answer.

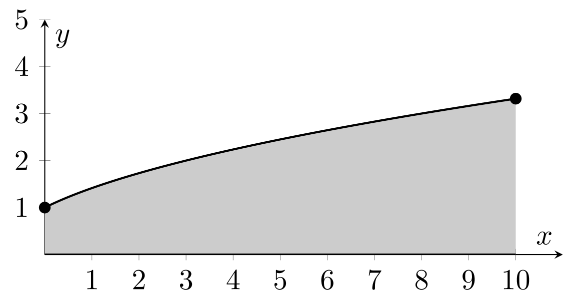
5. [7 points] Nzinga is going rock climbing at a local climbing gym. The gym building is shaped as follows. Its base is the triangular region shown in the figure below. The cross sections of the gym perpendicular to the y-axis are semicircles.



Write, but do not evaluate, an integral which gives the volume enclosed by the building.

6. [10 points] Denise and Trystan are undersea research scientists, and they are preparing to descend into the ocean in a newly-constructed submarine. The submarine's shape is given by rotating the region below the curve $y = \sqrt{x+1}$, above the x-axis, and between $x = 0$ and $x = 10$ (see figure) about the x-axis. Here, x and y are measured in meters.

Graph of $y = \sqrt{x+1}$ from $x = 0$ to $x = 10$



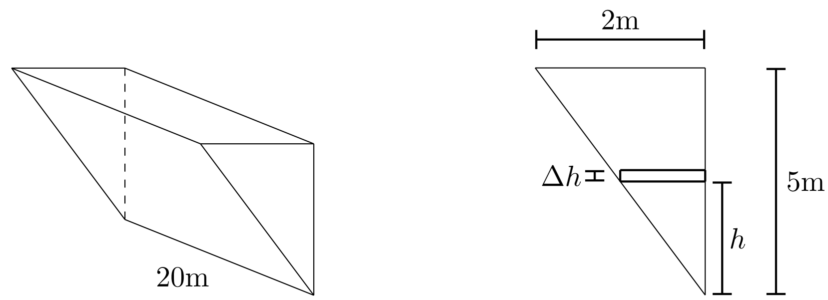
The density of the submarine is not constant, due to the advanced materials used in its construction. Instead, the density $p(x)$ varies, and is given by $p(x) = (x-5)^2 + 1$ kg/m^3.

a. [5 points] Write an expression for the **volume** of a slice of the submarine at position x and of thickness Δx. Include units.

b. [2 points] Write an expression for the **mass** of the slice you found in part (a). Include units.

c. [3 points] Write, but do not evaluate, an integral which gives the **total mass** of the submarine. Include units.

7. [13 points] A drinking water facility needs to pump water out of an underground tank. The tank is 20 meters in length with right-triangular cross-sections perpendicular to the ground as shown in the figure. The top of the tank is a 2m by 20m rectangle. The **top** of the tank lies **5 meters below the surface of the earth.** Recall that $g = 9.8\text{m/s}^2$, where g is the gravitational constant.



Underground Tank

a. [5 points] Write an expression for the **volume** (in cubic meters) of a horizontal rectangular slice of water at height h above the bottom of the tank, with thickness Δh. Your answer should not involve an integral.

b. [2 points] The density of water is approximately 1000 kg/m^3. Write an expression for the **weight** (in Newtons) of the slice of water from part (a). Your answer should not involve an integral.

c. [3 points] Write an expression for the **work** (in Joules) needed to pump the slice of water (from parts (a) and (b)) to the surface of the earth. Your answer should not involve an integral.

d. [3 points] Assuming the tank is initially full of water, write an integral for the **total work** (in Joules) needed to pump all of the water to the surface of the earth.

8. [12 points]

a. [6 points] Split the function

$$f(x) = \frac{x+2}{(x-2)^2(x-1)}$$

into partial fractions. Do not integrate your result. Please show all of your work.

8. (continued)

b. [6 points] Given the partial fraction decomposition

$$\frac{-3x}{(x+1)(x^2+1)} = \frac{3}{2(x+1)} - \frac{3(x+1)}{2(x^2+1)},$$

evaluate the following indefinite integral, showing all of your work:

$$\int \frac{-3x}{(x+1)(x^2+1)}\, dx.$$

9. [10 points]

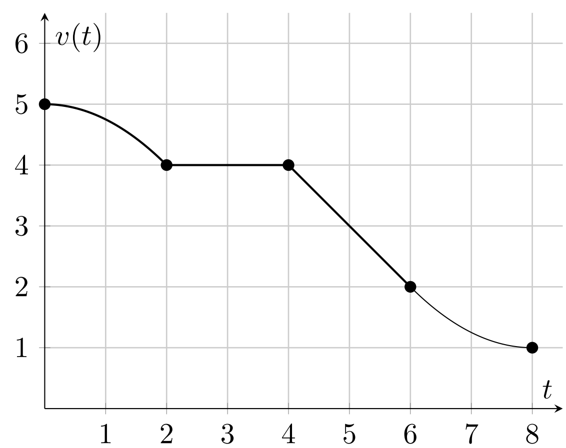
a. [6 points] As a particle moves in the xy plane, it traces out the curve $y = g(x)$ for $1 \leq x \leq 5$, where $g(x)$ is the function

$$g(x) = \int_{2x}^{x^4} \sin(t^3)\, dt + 100e^{\pi}.$$

Set up, but do not evaluate, an expression with a *single* integral which gives the arclength of the path of the particle. Your answer should not involve the letter g.

b. [4 points] A different particle is traveling with velocity given by $v(t)$ meters/second, where $v(t)$ is the function in the following graph: The units of t are seconds.

Graph of $y = v(t)$



Using a left Riemann sum with four subdivisions, estimate the distance the particle travels in the first 8 seconds of its journey. Is this an underestimate or an overestimate of the actual distance traveled in the first 8 seconds? Justify your answer.