Math 116 — Final Exam — December 12, 2025

EXAM SOLUTIONS

1. Please neatly write your 8-digit UMID number, your initials, your instructor's first and/or last name, and your section number in the spaces provided.
2. This exam has 12 pages including this cover.
3. There are 10 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it.
6. If you need more space to answer a question, please use the back of an exam page. Clearly indicate on your exam if you are using the back of a page, and also clearly label the problem number and part you are doing on the back of the page.
7. You are allowed notes written on two sides of a $3'' \times 5''$ note card. You are NOT allowed other resources, including, but not limited to, notes, calculators or other electronic devices.
8. For any graph or table that you use to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how you used the graph or table to find the answer.
9. Include units in your answer where that is appropriate.
10. Problems may ask for answers in *exact form.* Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is not.
11. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	9	
2	14	
3	5	
4	13	
5	10	

Problem	Points	Score
6	6	
7	10	
8	9	
9	11	
10	13	
Total	100	

1. [9 points] Some values of the function $g(x)$ and its derivative are given in the table below. Suppose that both $g(x)$ and $g'(x)$ are continuous.

x	1	3	5	7	9
$g(x)$	3	1	4	2	5
$g'(x)$	-2	0	3	-1	4

Using the information given above, find the following. Be sure to **show all of your work.** Your answers should not involve the letter g, but you **do not need to simplify them.**

a. [3 points] Suppose $F(x) = \int_1^{x^2} g(t)\, dt$. Find $F'(3)$.

Solution: By the Second Fundamental Theorem of Calculus, combined with the Chain Rule,

$$F'(x) = 2xg(x^2)$$

so

$$F'(3) = 2 \cdot 3 \cdot g(9) = 30.$$

Answer: 30

b. [3 points] Find $\lim_{x\to 1} \frac{3\ln(x) + g(x) - 3}{x-1}$.

Solution: We use L'Hospital's rule to find the limit.

$$\begin{aligned} \lim_{x\to 1} \frac{3\ln(x) + g(x) - 3}{x-1} &\overset{\text{LH}}{=} \lim_{x\to 1} \frac{3}{x} + g'(x) \\ &= 3 + g'(1) \\ &= 1 \end{aligned}$$

Answer: 1

c. [3 points] Use MID(2) to find the approximate value of $\int_1^9 \frac{g(x)}{1+x^3}\, dx$. Write out all the terms in your sum and do not attempt to simplify.

Solution: Note that the midpoints we use are 3 and 7. Then our MID(2) approximation is

$$4 \cdot \frac{g(3)}{1+3^3} + 4 \cdot \frac{g(7)}{1+7^3}.$$

Answer: $\frac{4}{1+3^3} + \frac{8}{1+7^3}$

2. [14 points] A function $f(x)$ is defined on the interval $(0,2)$ by its Taylor series around $x=1$,

$$f(x)=\sum_{n=0}^{\infty}\frac{1}{2n+1}(x-1)^{2n+1}.$$

a. [3 points] Find $f^{(2025)}(1)$ and $f^{(2026)}(1)$.

Solution: If we write down the Taylor series of f around 1, $f^{(2025)}(1)$ appears in the coefficient of $(x-1)^{2025}$, where $n=1012$. Therefore, comparing coefficients, we get

$$\frac{1}{2025}=\frac{f^{(2025)}(1)}{2025!},$$

and thus $f^{(2025)}(1)=\dfrac{2025!}{2025}=2024!$.
We also notice that all even powers of x have zero coefficient, so $f^{(2026)}(1)=0$.

Answer: $f^{(2025)}(1)=$ 2024! and $f^{(2026)}(1)=$ 0

b. [5 points] Find the degree 7 Taylor polynomial around $x=1$ for $f(x)$.

Solution: We include all the terms up to $(x-1)^7$. Therefore, the degree 7 Taylor polynomial is

$$(x-1)+\frac{1}{3}(x-1)^3+\frac{1}{5}(x-1)^5+\frac{1}{7}(x-1)^7.$$

Answer: $(x-1)+\frac{1}{3}(x-1)^3+\frac{1}{5}(x-1)^5+\frac{1}{7}(x-1)^7$

c. [3 points] Find the degree 6 Taylor polynomial around $x=1$ for $f'(x)$, the **derivative** of $f(x)$.

Solution: We take the derivative of the polynomial in **b.** and get

$$1+(x-1)^2+(x-1)^4+(x-1)^6.$$

Answer: $1+(x-1)^2+(x-1)^4+(x-1)^6$

d. [3 points] Find a closed-form expression of the derivative $f'(x)$ which applies on the interval $(0,2)$. Closed form means your answer should not include ellipses or sigma notation.

Solution: Using our known Taylor series, we notice that

$$f'(x)=\sum_{n=0}^{\infty}(x-1)^{2n}=\frac{1}{1-(x-1)^2}$$

on $(0,2)$, which is the interval of convergence of $f(x)$.

Answer: $f'(x)=\dfrac{1}{1-(x-1)^2}$

3. [5 points] Throughout this problem, suppose that:

- a_n is a sequence with $a_1 = 60$, and with $a_{n+1} = \frac{1}{3}a_n$ for all $n \geq 1$.
- b_n is a sequence with $b_1 = \frac{1}{2}$, and with $b_{n+1} = 2b_n$ for all $n \geq 1$.
- $S_n = \sum_{j=1}^{n} a_j$ and $R_n = \sum_{k=1}^{n} S_k$.

For each of the following sequences, determine whether the sequence converges or diverges, and if it converges, determine the value that it converges to. Justification is not required.

a. [1 point] a_n

Solution: Since $a_{n+1} = \frac{1}{3}a_n$, we see that $a_n = 60\left(\frac{1}{3}\right)^{n-1}$ and thus $\lim_{n\to\infty} a_n = 0$.

Circle one: **Diverges** $\boxed{\textbf{Converges to } \underline{\quad 0 \quad}}$

b. [1 point] b_n

Solution: Using a similar argument as **a.** we have, $b_n = \frac{2^{n-1}}{2}$, from which we can see that b_n diverges.

Circle one: $\boxed{\textbf{Diverges}}$ **Converges to** ______________

c. [1 point] $c_n = a_n \cdot b_n$

Solution: Using **a.** and **b.** we can compute that $c_n = a_n \cdot b_n = 30\left(\frac{2}{3}\right)^{n-1}$ and thus $\lim_{n\to\infty} c_n = 0$.

Circle one: **Diverges** $\boxed{\textbf{Converges to } \underline{\quad 0 \quad}}$

d. [1 point] S_n

Solution: Note that S_n is a finite geometric series, and so

$$\lim_{n\to\infty} S_n = \sum_{n=1}^{\infty} 60\left(\frac{1}{3}\right)^{n-1} = 60\left(\frac{1}{1-\frac{1}{3}}\right) = 90.$$

Circle one: **Diverges** $\boxed{\textbf{Converges to } \underline{\quad 90 \quad}}$

e. [1 point] R_n

Solution: Since $R_n = \sum_{k=1}^{n} S_k$ and $\lim_{n\to\infty} S_n = 90 \neq 0$, the nth term test for divergence tells us that R_n diverges.

Circle one: $\boxed{\textbf{Diverges}}$ **Converges to** ______________

4. [13 points] A vertical climbing wall is described in coordinates, where (x, y) is the position x meters to the right of a central podium and y meters above the floor. Suppose two climbers, Sara and Tina, begin climbing the wall at the same time. Their positions t minutes after they start climbing are given by:

$$\text{Sara: } \begin{cases} x(t) = -(t-1)^2 \\ y(t) = 3t, \end{cases} \qquad \text{Tina: } \begin{cases} x(t) = 1 + \cos(\pi t) \\ y(t) = t^2 + 2. \end{cases}$$

a. [4 points] Tina and Sara bumped into each other one time during the process of climbing. At what time did this happen? Justify your answer.

Solution: They bump into each other if their x and y-coordinates are both the same. If we equate the y coordinates, we get $3t = t^2 + 2$ and solving gives us $t = 1$ and $t = 2$. Plugging in $t = 1$ and $t = 2$ to the x-coordinate parameterizations we note that the x coordinates of Sara and Tina are the same at $t = 1$. Therefore, they bumped into each other at $t = 1$.

Answer: $t =$ 1

b. [2 points] After Sara and Tina bumped into each other, they climbed for some further distance and eventually crossed the finish line, which is a horizontal line, at the same time. When did they cross the finish line, and what is the equation of the finish line?

Solution: Crossing the finish line at the same time means that Sara and Tina's y-coordinates are the same at that time. From part **a.**, we know this happens at $t = 1$ and $t = 2$. We already know from **a.** that they bumped into each other at $t = 1$, so $t = 2$ must be the time at which they both crossed the finish line. We plug in $t = 2$ to either Sara or Tina's y-coordinates and get $y = 6$.

Answer: They crossed the finish line at $t =$ 2

Answer: The equation of the finish line is $y =$ 6

c. [4 points] Write an expression involving one or more integrals that gives the total distance traveled by Tina from the time she starting climbing until she crossed the finish line. Do not evaluate your integral(s). Show your work.

Solution: For Tina, $x'(t) = -\pi \sin(\pi t)$ and $y'(t) = 2t$, so the total distance traveled is

$$\int_0^2 \sqrt{x'(t)^2 + y'(t)^2}\, dt = \int_0^2 \sqrt{\pi^2 \sin^2(\pi t) + 4t^2}\, dt.$$

Answer: $\int_0^2 \sqrt{\pi^2 \sin^2(\pi t) + 4t^2}\, dt$

d. [3 points] What is Tina's speed at $t = \frac{1}{2}$? **Include units**. You do not need to simplify your answer but you should show your work.

Solution: Tina's speed at time t is $\sqrt{\pi^2 \sin^2(\pi t) + 4t^2}$ m/min, so plugging in $t = \frac{1}{2}$ we get $\sqrt{\pi^2 + 1}$ m/min .

Answer: $\sqrt{\pi^2 + 1}$ **Units:** m/min

5. [10 points] Consider the Taylor series, $\displaystyle\sum_{n=1}^{\infty} \frac{9n^2+8}{2^n \cdot (n^3+1)}(x+3)^n$.

a. [1 point] Determine the center, a, of the Taylor series.

Answer: $a =$ -3

b. [9 points] The radius of convergence of the Taylor series is 2 (you do **not** need to show this). Determine the interval of convergence of the Taylor series. Show all your work, including full justification for series behavior.

Solution: Given that the center is $x=-3$ and that the radius of convergence is 2, we need to check the endpoints $x=-1$ and $x=-5$.
When $x=-1$, the series is

$$\sum_{n=1}^{\infty} \frac{9n^2+8}{2^n \cdot (n^3+1)} 2^n = \sum_{n=1}^{\infty} \frac{9n^2+8}{n^3+1}.$$

We can use the Limit Comparison Test (LCT) or Direct Comparison Test (DCT) to show divergence. For the Limit Comparison Test (LCT), we note that

$$\lim_{x\to\infty} \frac{\frac{9n^2+8}{n^3+1}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{9n^3+8n}{n^3+1} = 9 > 0,$$

and that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (by the p-test with $p=1$). Therefore, by the LCT, $\displaystyle\sum_{n=1}^{\infty} \frac{9n^2+8}{n^3+1}$ diverges.
Alternatively, for the Direct Comparison Test (DCT), we note that for $n \geq 1$,

$$\frac{9n^2+8}{n^3+1} > \frac{9n^2}{n^3+1} \geq \frac{9n^2}{2n^3} = \frac{9}{2n},$$

and that $\displaystyle\sum_{n=1}^{\infty} \frac{9}{2n}$ diverges (by the p-test with $p=1$). Therefore, by the DCT, $\displaystyle\sum_{n=1}^{\infty} \frac{9n^2+8}{n^3+1}$ diverges.
When $x=-5$, the Taylor series is

$$\sum_{n=1}^{\infty} \frac{9n^2+8}{2^n \cdot (n^3+1)} (-2)^n = \sum_{n=1}^{\infty} (-1)^n \frac{9n^2+8}{n^3+1}.$$

We want to use Alternating Series Test (AST) to justify convergence. We see that

1. $\displaystyle\lim_{n\to\infty} \frac{9n^2+8}{n^3+1} = 0$, and
2. $\displaystyle\frac{9n^2+8}{n^3+1}$ is monotone decreasing for $n \geq 1$.

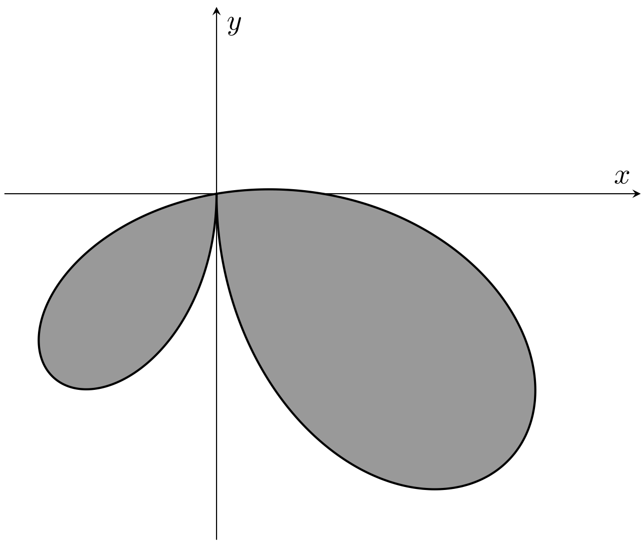
so the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{9n^2+8}{n^3+1}$$

converges by alternating series test (AST). Hence the interval of convergence is $[-5,-1)$.

Answer: The interval of convergence is $[-5,-1)$

6. [6 points] A graphic designer makes posters for David, a singer who famously used to have a unique haircut. In a poster for an old album, the boundary of David's haircut is given by the polar curve $r(\theta) = \cos(\theta) + 3\sin(2\theta)$, where $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$. A graph of the curve is shown below, and David's hair is represented by the shaded region.



a. [2 points] What are the x- and y-coordinates of the curve in terms of θ?

Solution: We have

$$x(\theta) = r(\theta)\cos(\theta) = \cos^2(\theta) + 3\sin(2\theta)\cos(\theta)$$
$$y(\theta) = r(\theta)\sin(\theta) = \cos(\theta)\sin(\theta) + 3\sin(2\theta)\sin(\theta)$$

Answer: $x(\theta) =$ $\cos^2(\theta) + 3\sin(2\theta)\cos\theta$

Answer: $y(\theta) =$ $\cos(\theta)\sin(\theta) + 3\sin(2\theta)\sin(\theta)$

b. [2 points] Find $\dfrac{dx}{d\theta}$ in terms of θ.

Solution: Using chain rule and product rule, we have

$$\frac{dx}{d\theta} = -2\cos(\theta)\sin(\theta) + 6\cos(2\theta)\cos(\theta) - 3\sin(2\theta)\sin(\theta).$$

Answer: $\dfrac{dx}{d\theta} =$ $-2\cos(\theta)\sin(\theta) + 6\cos(2\theta)\cos(\theta) - 3\sin(2\theta)\sin(\theta)$

c. [2 points] At which of the following values of θ could the above curve have a vertical tangent line? Circle all options that apply.

Solution: We look at values of θ such that $\dfrac{dx}{d\theta} = 0$, and can see that $\theta = \pi/2$ and $\theta = 3\pi/2$ satisfy the requirement.

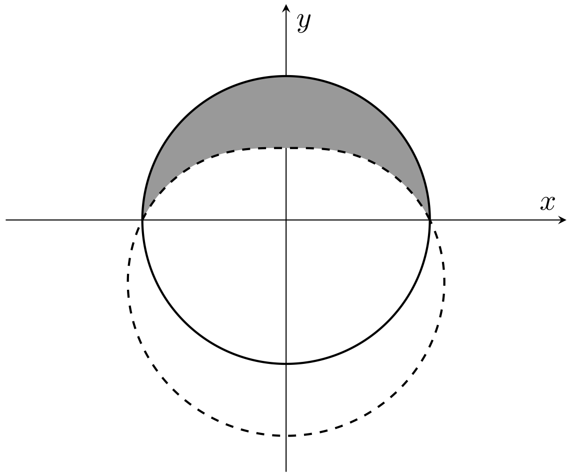
i. $\boxed{\theta = \pi/2}$

ii. $\theta = \pi$

iii. $\boxed{\theta = 3\pi/2}$

iv. None of the above

7. [10 points] Nowadays, David's haircut is more simplistic. In a poster for a more recent album, David's haircut is given by the shaded region below. The solid curve is a circle centered at the origin with radius 2, while the dashed curve is given by the equation $r(\theta) = 2 - \sin(\theta)$, where $0 \le \theta \le 2\pi$.



a. [2 points] Find the two intersection points of the two curves in the diagram. Express the intersection points in polar coordinate form (r, θ), where $r > 0$ and $0 \le \theta < 2\pi$.

Solution: The intersection points correspond to $2 - \sin(\theta) = 2$. Solving, we find $\theta = 0$, and $\theta = \pi$ since we require $0 \le \theta < 2\pi$.

Answer: $(2, 0)$ and $(2, \pi)$

b. [4 points] Write an expression involving one or more integrals that gives the total perimeter of the shaded region. Your final answer should not involve the letter r. Do not evaluate your integral(s).

Solution: The solid line is the arc of a semicircle with radius 2, so has length 2π. For the dotted curve, we note that $r'(\theta) = -\cos(\theta)$, so the arclength is

$$\int_0^{\pi} \sqrt{r(\theta)^2 + r'(\theta)^2}\, d\theta = \int_0^{\pi} \sqrt{(2 - \sin\theta)^2 + \cos^2\theta}\, d\theta.$$

Therefore the total perimeter of the shaded area is

$$2\pi + \int_0^{\pi} \sqrt{(2 - \sin\theta)^2 + \cos^2\theta}\, d\theta.$$

Answer: $2\pi + \int_0^{\pi} \sqrt{(2 - \sin\theta)^2 + \cos^2\theta}\, d\theta$

c. [4 points] Write an expression involving one or more integrals that gives the area of the shaded region. Your final answer should not involve the letter r. Do not evaluate your integral(s).

Solution:
The area inside the dotted curve but above the x-axis is given by $\frac{1}{2}\int_0^{\pi} (2 - \sin\theta)^2\, d\theta$. We subtract this from the area of the semi-circle above the x-axis to obtain

$$2\pi - \frac{1}{2}\int_0^{\pi} (2 - \sin\theta)^2\, d\theta.$$

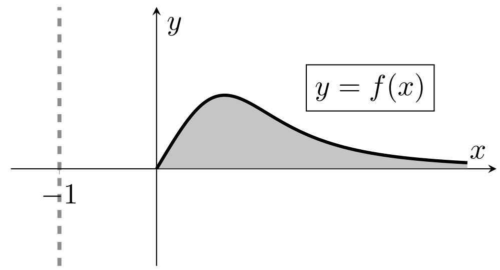
Answer: $2\pi - \frac{1}{2}\int_0^{\pi} (2 - \sin\theta)^2\, d\theta$

8. [9 points] The parts of this problem are unrelated.

a. [4 points]
Consider the infinite region in the first quadrant between the x-axis and the curve $y = f(x)$ where

$$f(x) = \frac{\arctan(x)}{x^3 + 1},$$

as shaded in the figure to the right.

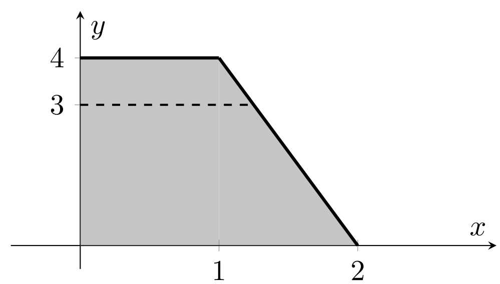


Find an expression involving one or more integrals for the volume of the solid of revolution given by rotating the infinite shaded region around the line $x = -1$. Your expression should not contain the letter f. Do not evaluate your integral(s).

Solution: Using the shell method, we see that the volume is

$$\int_0^\infty 2\pi(x+1)f(x)\ dx = \int_0^\infty 2\pi(x+1)\frac{\arctan(x)}{x^3+1}\ dx.$$

b. [5 points]
Consider the region in the first quadrant bounded by the y-axis, the x-axis, the line $y = 4$, and the line $y = 8 - 4x$, as shaded in the figure to the right.
Assume that all distances are in meters. You may assume that the acceleration due to gravity is $g = 9.8\text{m/s}^2$.



A tank is formed by rotating this region around the y-axis. The tank is partially filled with a liquid, to a depth of 3 meters from the bottom of the tank (indicated by the dashed line in the figure above). Suppose the liquid has density $p(y) = (1+y)^{1/3}$ kg/m^3 where y, in meters, is the height of a point above the bottom of the tank. Find an expression involving one or more integrals for the work, in Joules, needed to pump all the liquid to the top of the tank. Your answer should not involve the letter p. Do not evaluate your integral(s).

Solution: If we consider a horizontal slice at height y from the bottom having thickness Δy, the horizontal distance from the y axis is $x = \dfrac{8-y}{4}$ m, so the volume of that slice is

$$\pi\left(\frac{8-y}{4}\right)^2 \Delta y \quad \text{m}^3, \quad \text{so its weight is} \quad 9.8\pi\left(\frac{8-y}{4}\right)^2 (1+y)^{1/3}\Delta y \quad \text{kg}.$$

Therefore the work required to pull the slice out of the tank is

$$9.8\pi\left(\frac{8-y}{4}\right)^2 (1+y)^{1/3}(4-y)\Delta y \quad \text{Joules},$$

and the total amount of work is

$$\int_0^3 9.8\pi\left(\frac{8-y}{4}\right)^2 (1+y)^{1/3}(4-y)\ dy \quad \text{Joules}.$$

9. [11 points] Consider the function

$$g(x) = \frac{4}{\sqrt{1+5x^3}}.$$

a. [5 points] Give the first three nonzero terms of the Taylor series of $g(x)$ centered about $x = 0$. Show all your work.

Solution: We use the binomial series

$$(1+y)^{-1/2} = 1 - \frac{1}{2}y + \frac{3}{8}y^2 + \cdots$$

where we replace y with $5x^3$, and multiply by 4. We find that the first three terms of the Taylor series of $g(x)$ centered at 0 are:

$$4 - \frac{4}{2}(5x^3) + \frac{12}{8}(5x^3)^2 = 4 - 10x + \frac{75}{2}x^6.$$

Answer: $4 - 10x^3 + \frac{75}{2}x^6$

b. [2 points] What is the radius of convergence of the Taylor series for $g(x)$ around $x = 0$?

Solution: The binomial series converges for $|y| < 1$, so to figure out the radius of convergence of the Taylor series for $g(x)$, we look for where

$$|5x^3| < 1$$

which gives us

$$|x| < \left(\frac{1}{5}\right)^{1/3}.$$

Answer: $\left(\frac{1}{5}\right)^{1/3}$

c. [4 points] Suppose that $G(x)$ is an antiderivative for $g(x)$ which satisfies $G(0) = 5$. Give the first four non-zero terms of the Taylor series for $G(x)$ centered about $x = 0$.

Solution: We use the first three non-zero terms of the Taylor series of $g(x)$ we got in **a.** Taking the indefinite integral, we get

$$C + 4x - \frac{10}{4}x^4 + \frac{75}{14}x^7.$$

Since $G(0) = 5$, we get that the first four non-zero terms of the Taylor series of $G(x)$ are

$$5 + 4x - \frac{5}{2}x^4 + \frac{75}{14}x^7.$$

Answer: $5 + 4x - \frac{5}{2}x^4 + \frac{75}{14}x^7$

10. [13 points]

a. [4 points] Suppose that $f(x)$ is a positive, decreasing, differentiable function which is defined for all real numbers, and that for $n \geq 1$, a_n is given by the LEFT(n) approximation to the integral $\int_1^{n+1} f(x)\, dx$. If $\lim\limits_{n\to\infty} a_n = 23$, then which of the following must be true? Circle **all** options which apply.

i. $\int_1^\infty f(x)\, dx$ converges to 23.

ii. $\int_1^\infty f(x)\, dx$ converges to a number greater than 23.

iii. $\boxed{\int_1^\infty f(x)\, dx \text{ converges to a number less than 23.}}$

iv. $\int_1^\infty f(x)\, dx$ could diverge.

v. $\boxed{\sum_{n=1}^{\infty} f(n) \text{ converges to 23.}}$

vi. $\sum_{n=1}^{\infty} f(n)$ converges to a number greater than 23.

vii. $\sum_{n=1}^{\infty} f(n)$ converges to a number less than 23.

viii. $\sum_{n=1}^{\infty} f(n)$ could diverge.

b. [2 points] Suppose that $f(h)$ is a probability density function for h, the maximum depth, in hundreds of meters, that a king penguin reaches on a single dive to catch fish. Which of the following statements is best supported by the equation $f(1) = 0.5$? Circle the **one** best answer.

i. There is about a 50% chance that a king penguin dives to maximum depth of 100 meters in any given dive.

ii. There is roughly a 100% chance that a king penguin will dive to a maximum depth of less than 50 meters in any given dive.

iii. If a king penguin were to dive for fish 200 times, the maximum depth of about 20 of those dives would be between 100 and 120 meters.

iv. Approximately 0.5% of all of a king penguin's dives will be to a maximum depth of between 50 and 150 meters.

v. The median maximum depth that king penguins dive to is 100 meters.

vi. NONE OF THESE

c. [4 points] Which of the following series converge? Circle **all** options that apply.

i. $\boxed{\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}}$

ii. $\boxed{\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n)}{n^3}}$

iii. $\boxed{\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{n!}}$

iv. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{2n}}{n^{2/3}}$

v. $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n\ln(n)}$

vi. NONE OF THESE

d. [3 points] Which of the following series converge to 50? Circle **all** options that apply.

i. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 50}{(2n)!}\left(\frac{\pi}{2}\right)^{2n}$

ii. $\boxed{\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 50}{(2n+1)!}\left(\frac{\pi}{2}\right)^{2n+1}}$

iii. $\boxed{\displaystyle\sum_{n=0}^{\infty} \frac{(\ln(50))^n}{n!}}$

iv. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}\left(e^{50}-1\right)^n$

v. NONE OF THESE

"Known" Taylor series (all around $x = 0$):

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \cdots + \frac{(-1)^n\, x^{2n+1}}{(2n+1)!} + \cdots \qquad \text{for all values of } x$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \cdots + \frac{(-1)^n\, x^{2n}}{(2n)!} + \cdots \qquad \text{for all values of } x$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots \qquad \text{for all values of } x$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + \frac{(-1)^{n+1} x^n}{n} + \cdots \qquad \text{for } -1 < x \le 1$$

$$(1+x)^p = 1 + px + \frac{p(p-1)}{2!}\, x^2 + \frac{p(p-1)(p-2)}{3!}\, x^3 + \cdots \qquad \text{for } -1 < x < 1$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots \qquad \text{for } -1 < x < 1$$

Select Values of Trigonometric Functions:

θ	$\sin\theta$	$\cos\theta$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$