MATH 116 — EXAM I

Department of Mathematics
University of Michigan

February 6, 2003

NAME: ____________________ ID NUMBER: ____________________

SIGNATURE: ____________________

INSTRUCTOR: ____________________ SECTION NO: ____________________

1. This exam has nine pages including this cover. There are ten questions.
2. Use of books, notes, or scratch paper is **NOT** allowed. You may certainly use your calculator (but not its manual). One 3x5-inch notecard is allowed.
3. **Show all of your work!** Partial credit is available for many problems but can only be given if the graders understand your work. Be sure to explain your reasoning carefully. If you are basing your reasoning on a graph, then sketch the graph. Include units in your answers whenever appropriate.
4. One of the skills being tested in this exam is your ability to interpret detailed, precisely worded directions. Be sure to read the directions carefully and do all that is asked.
5. Stay calm.

PROBLEM	POINTS	SCORE
1	12	
2	6	
3	12	
4	6	
5	10	
6	10	
7	10	
8	10	
9	14	
10	10	
TOTAL	100	

1. (3 pts each) Circle true or false. No explanation necessary.

True or **False**: If $0 < f(x) < g(x)$ for all x, then $\int_1^4 \frac{g(x)}{f(x)} dx > 3$.

True or **False**: $\int_\pi^\pi \frac{\sin^2(x^5) - \cos(42x)}{x^2+x+1} dx > 2$.

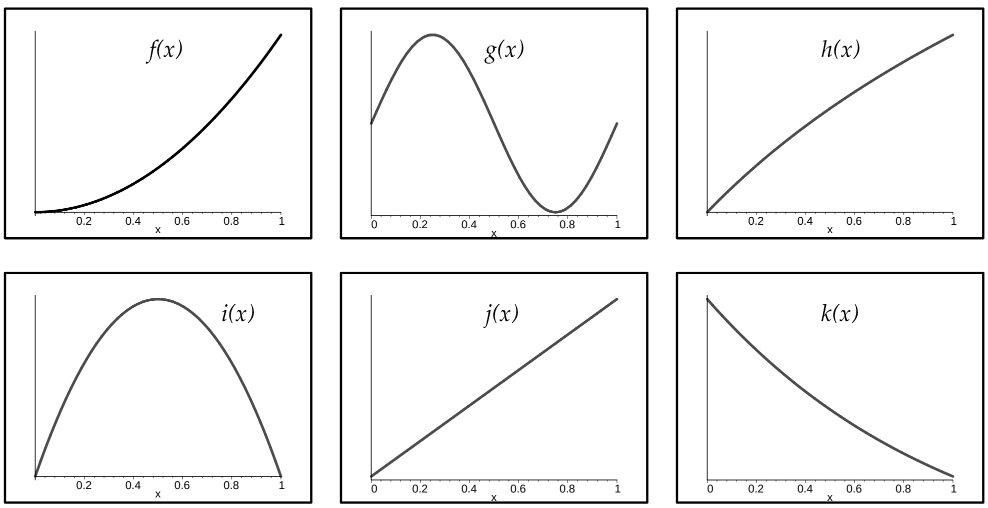
True or **False**: $\int_1^\infty \frac{1}{x^{71}} dx$ converges.

True or **False**: If $g(x) + 2$ is an antiderivative of $f(x)$, then $g(x)$ is also an antiderivative of $f(x)$.

2: (6 pts) State the construction theorem for antiderivatives, also known as the second fundamental theorem of Calculus. Your answer should be a complete statement.

3: (12 pts) Below are graphs of several functions $f(x)$, $g(x)$, $h(x)$, $i(x)$, $j(x)$, and $k(x)$. Do not assume that the y-axis scales on these graphs are equal or even comparable. We have calculated LEFT(6), RIGHT(6), TRAP(6), and MID(6) for four of these six functions. Label each column with the name of the function estimated in that column. Of course, not every function label will be used! No explanation necessary.

Function:				
LEFT(6):	64.2	.328	.255	80.0
RIGHT(6):	65.8	.444	.421	80.0
TRAP(6):	65.0	.386	.338	80.0
MID(6):	65.0	.388	.331	80.0



4: (6 pts) Write an integration problem (of your choice) for which the substitution $w = 1/x$ would be the best way to start. You need not evaluate your own integral.

Your integral: __________________________

5: (10 pts) Does $\int_0^8 \frac{5+\sin(x)}{x(8+\cos(x))}dx$ converge or diverge? Demonstrate unequivocally that your answer is correct.

6: (10 pts) A common entry in many integral tables is:

$$\int \cos^n(x)dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x dx.$$

Show how to use this information to calculate $\int \cos^4(x)dx$. For full credit, show all your work.

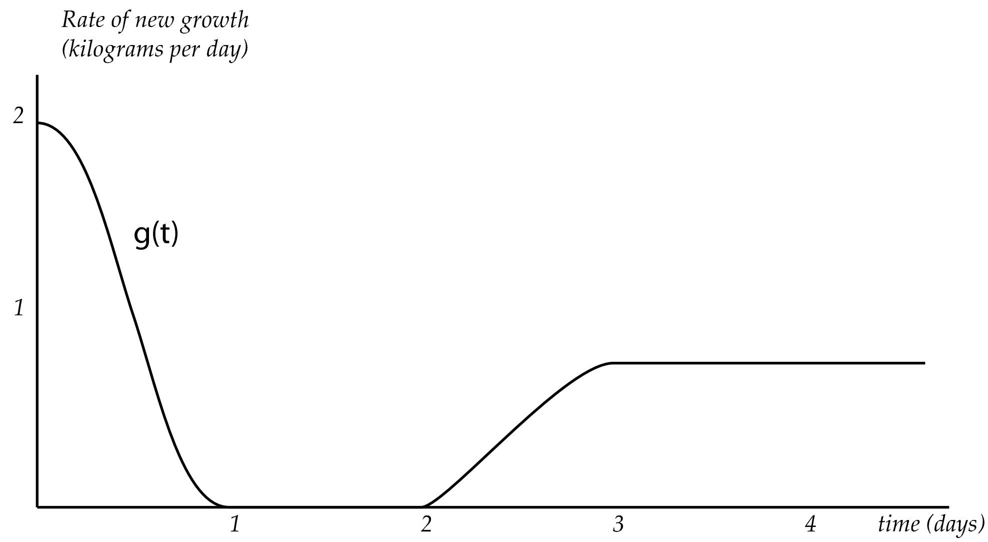
7: (10 pts) Using integration by parts, calculate

$$\int e^{-x} cos(x) dx.$$

8: (10 pts) Calculate the exact value of this definite integral. You will be graded on the correctness of your work, so show it carefully.

$$\int_{-\infty}^{0} \frac{e^x}{1+e^x} dx$$

9: (14 pts) In springtime, as a nameless old tree quietly builds its leaves and branches by drawing matter out of the air and earth, an unnamed old botanist measures the process with care:



a. (2 pts) Estimate $\int_0^4 g(t)dt$.

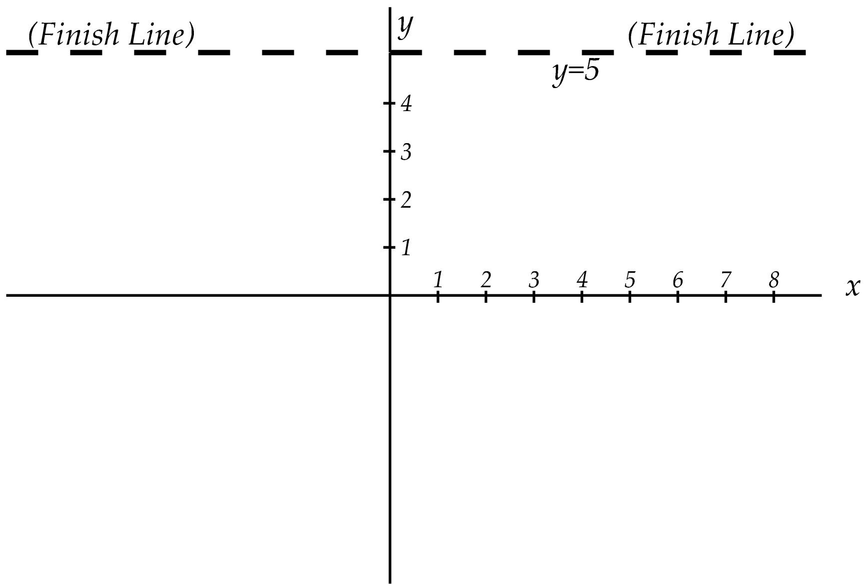
b. (4 pts) Explain what your answer to part "a" tells you about the tree.

c. (6 pts) Sketch a possible graph of the mass of the tree as a function of time during this particular season. Label your graph carefully.

d. (2 pts) What can you say about the derivative of the function sketched in part c? Answer *briefly*.

10: (10 pts) The newest FOX reality show, "BattleBugs: Clash of the Beetles" begins (at $t = 0$) with eight assorted insects placed randomly on a large mat (the "battlefield", pictured below), on which is marked a "finish line." The producers hoped that the bugs would battle to be first to cross the finish line, but instead they just wander around, each according to its nature. The motion of each bug is described by the equations below. Both x and y are measured in inches.

The Battlefield



Hercules Beetle $x(t) = \cos(t/2)$ $y(t) = \sin(t/2)$	Ladybug $x(t) = e^{-t}$ $y(t) = e^{-2t}$	Tiger Beetle $x(t) = 1 + t$ $y(t) = -1 + 8t$	Longhorned Beetle $x(t) = 3 + t$ $y(t) = 4 - t$
Dung Beetle $x(t) = t$ $y(t) = -2$	Scarab $x(t) = 2 - 7t$ $y(t) = -1 - 7t$	June Beetle $x(t) = 0$ $y(t) = -1$	African Ground Beetle $x(t) = \sin(t)$ $y(t) = \cos(t)$

Which bug (or bugs) ...

a. move repetitively?

b. move fastest?

c. begin closest to the finish line?

d. will reach the finish line first?

e. will move very slowly (or not at all), in the long run?