MATH 116 — EXAM II

Department of Mathematics
University of Michigan

March 18, 2003

NAME: _____ ID NUMBER: _____

SIGNATURE: _____

INSTRUCTOR: _____ SECTION NO: _____

1. This exam has nine pages including this cover. There are nine questions.
2. Use of books, notes, or scratch paper is **NOT** allowed. You may certainly use your calculator (but not its manual). One 3x5-inch notecard is allowed.
3. **Show all of your work!** Partial credit is available for many problems but can only be given if the graders understand your work. Be sure to explain your reasoning carefully. If you are basing your reasoning on a graph, then sketch the graph. Include units in your answers whenever appropriate.
4. One of the skills being tested in this exam is your ability to interpret detailed, precisely worded directions. Be sure to read the directions carefully and do all that is asked.
5. Stay calm.

PROBLEM	POINTS	SCORE
1	6	
2	8	
3	12	
4	10	
5	18	
6	10	
7	16	
8	10	
9	10	
TOTAL	100	

1. (2 pts each) Circle true or false. No explanation necessary.

True or **False**: The Taylor series centered at $x = 2$ of e^x is

$$1 + (x-2) + \frac{(x-2)^2}{2!} + \frac{(x-2)^3}{3!} + \frac{(x-2)^4}{4!} + \frac{(x-2)^5}{5!} + \dots$$

True or **False**: If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=0}^{\infty} a_n$ converges.

True or **False**: The fourth derivative of

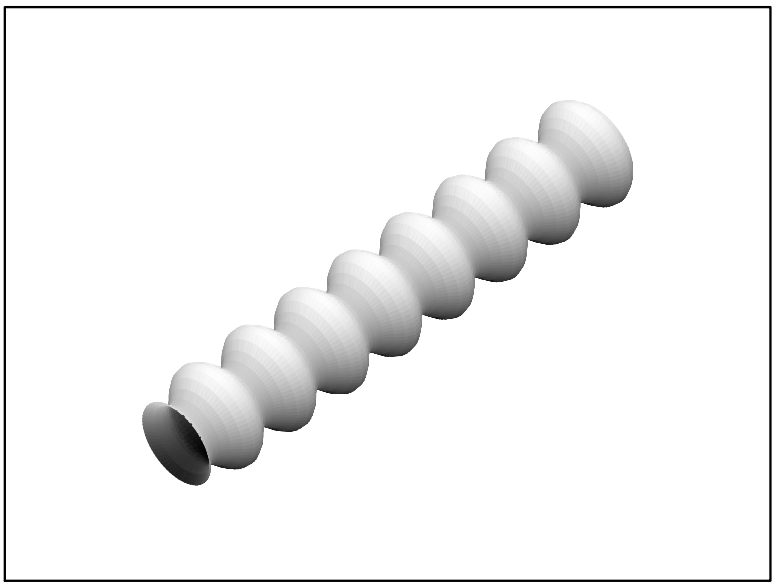
$$f(x) = 6 + 6x + 6x^2 + 6x^3 + \frac{x^4}{4} + 6x^5 + 6x^6 + 6x^7 + \dots$$

at the point $x = 0$ is 6.

2. (8 pts) Art inspires art: Whenever artists create art, other people have the opportunity to take inspiration and create more art. These responses can inspire still other works, and so on.

Assume that creating any number x of artistic works will inspire $.8x$ more artistic works (fractional works are ok–don't round). If you produce 75 sculptures, for how many total works can you claim indirect influence? In other words, how many artistic works are there, in total, which are part of your collection, inspired by your collection, inspired by those inspired by your collection, etc? Show your work carefully. Do not round, and do not estimate.

3. (12 pts) A decorative table leg (see diagram) is manufactured so that it is the volume of rotation of the function $f(x) = 4 + \sin(x)$ between $x = 0$ and $x = 16\pi$.



a. (8 pts) What is the volume of the table leg? Show all work, but (on this problem only) you may evaluate any integrals on your calculator.

b. (4 pts) Here's a plausible shortcut: Replace the complicated shape with a cylinder with height 16π and radius 4 (because the average radius above is 4), and apply the volume formula for a cylinder. Is the shortcut valid? Explain briefly.

4. (10 pts) A sphere has uniform density ρ and radius 5. Of course, its center of mass is located at the center of the sphere. Assume now that the left half is cut off and removed. What is the center of mass of the remaining right hemisphere?

5. (18 pts) Fred likes to juggle. So does jason. The number of minutes Fred can juggle five balls without dropping one is a random variable, with probability density function $f(t) = 0.8e^{-0.8t}$. Similarly, the function $j(t) = 1.5e^{-1.5t}$ describes jason's skill. Here t is time *in minutes*.

a. (2 pts) Find $\int_0^\infty f(t)dt$. No need to show work.

b. (5 pts) What percentage of jason's juggling attempts are "embarrassing," meaning they last for 10 seconds or less? Show your work.

c. (6 pts) How long can Fred juggle, on average? Show your work.

d. (5 pts) Who is the better juggler? Give a good reason for your decision.

6. (10 pts) Does $\frac{1}{3\ln(3)} + \frac{1}{4\ln(4)} + \frac{1}{5\ln(5)} + \frac{1}{6\ln(6)} + \ldots$ converge or diverge? Demonstrate unequivocally that your answer is correct.

7. (16 pts)

a. (8 pts) Find the Taylor series expansion for the function $\ln(2+x)$ centered at the point $x = 0$.

b. (4 pts) Using your calculator, graphically approximate the domain of convergence of this Taylor series. Accurately sketch a graph which suggests how you got your answer.

c. (4 pts) It is claimed "One way to approximate $\ln(10)$ is to plug in 8 to the series above, using the first dozen, hundred, or even more terms. The more terms you take, the better your approximation of $\ln(10)$." Explain why this plan will (or will not) work.

8. (10 pts) On this problem you must show your work and use exact methods. That is, calculator approximations are insufficient.

Find two values of x for which

$$x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \frac{x^{18}}{9!} - \frac{x^{22}}{11!} + \ldots = 1.$$

9. (10 pts) You have been offered a deferred student loan: You will be paid $\$10,000$ today, but after four years elapse you must begin a continuous repayment stream at a rate of \$2000 per year for five years. Assume all moneys earn continuous 6% interest.

You have concocted the following (ethically questionable) plan:

1. Take the cash.
2. Put the cash in the bank, earning 6% continuous interest.
3. In four years, begin repayment, as scheduled. Continue repayment for five years.

a. (4 pts) What is the future value ("future" means nine years later, when the entire loan is finally repaid) of the payment stream with which you pay back the loan?

b. (4 pts) What is the future value of the $\$10,000$ received today?

c. (2 pts) If you implement the plan above, how much "profit" will you have made in nine years?