MATH 116 — Final Exam

DEPARTMENT OF MATHEMATICS
University of Michigan

April 21, 2003

NAME: ______________________________ ID NUMBER: ____________________________

SIGNATURE: _______________________

INSTRUCTOR: _____________________ SECTION NO: ____________________________

1. This exam has eleven pages including this cover. There are eleven questions.
2. Use of books, notes, or scratch paper is **NOT** allowed. You may certainly use your calculator (but not its manual). One 3x5-inch notecard is allowed.
3. **Show all of your work!** Partial credit is available for many problems but can only be given if the graders understand your work. Be sure to explain your reasoning carefully. If you are basing your reasoning on a graph, then sketch the graph. Include units in your answers whenever appropriate.
4. One of the skills being tested in this exam is your ability to interpret detailed, precisely worded directions. Be sure to read the directions carefully and do all that is asked.
5. Stay calm.

PROBLEM	POINTS	SCORE
1	10	
2	10	
3	13	
4	13	
5	6	
6	6	
7	9	
8	8	
9	6	
10	11	
11	8	
TOTAL	100	

1. (2 pts each) Circle true or false. No explanation necessary.

True or **False**: If f is a continuous function, then $\int_0^9 f(x)dx$ is between LEFT(17) and RIGHT(17).

True or **False**: The sum $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \ldots$ diverges.

True or **False**: Applying separation of variables to a differential equation is always more accurate than using Euler's method.

True or **False**: Let $p(x)$ be a probability density function. Then for all x, $0 \leq p(x) \leq 1$.

True or **False**: If a metal rod has variable density $\rho(x)$ kilograms per meter, then its mass is the product of its length and the integral of $\rho(x)$.

2. (10 pts)

a. (5 pts) Write a single differential equation, all of whose solutions approach the value 60 in the long run.

b. (1 pt) Write another differential equation with the same property.

c. (4 pts) Write a general solution to your differential equation from part a.

3. (13 pts) The Altairian slime mold's rate of growth is inversely proportional to its weight. A culture which four days ago weighed only 3 grams has already grown to 5 grams.

a. (10 pts) Using a differential equation, find a formula for the slime mold's weight as a function of time. **Show all of your work.**

b. (3 pts) Calculate the slime mold's age. (It begins its life as a microscopic spore.)

4. (13 pts) In 1970 the worldwide black rhinoceros population was approximately 65,000. Assume that until 1970 the population was at the natural equilibrium population of this logistic differential equation:

$$dR/dt = .12R - \frac{R^2}{540,000}$$

But around 1970 demand for rhinoceros horn[1] dramatically increased poaching...

a. (4 pts) A scientist claims "Each year poachers now take 10% of the population!" If this is true, how should the differential equation be changed to reflect this fact? *Is it still logistic?*

b. (3 pts) If the scientist is correct, what will happen to the rhino population in the long run? No explanation necessary, but answer clearly.

c. (6 pts) As of 2003, there are approximately 2,500 black rhinos left. If poaching can be eradicated, so that the population again follows the original logistic model, when will the population reach 50,000? Show all work.

[1] for dagger handles in North Yemen and folk medicine in Asia

5. (6 pts) Find all equilibrium solutions of the equation

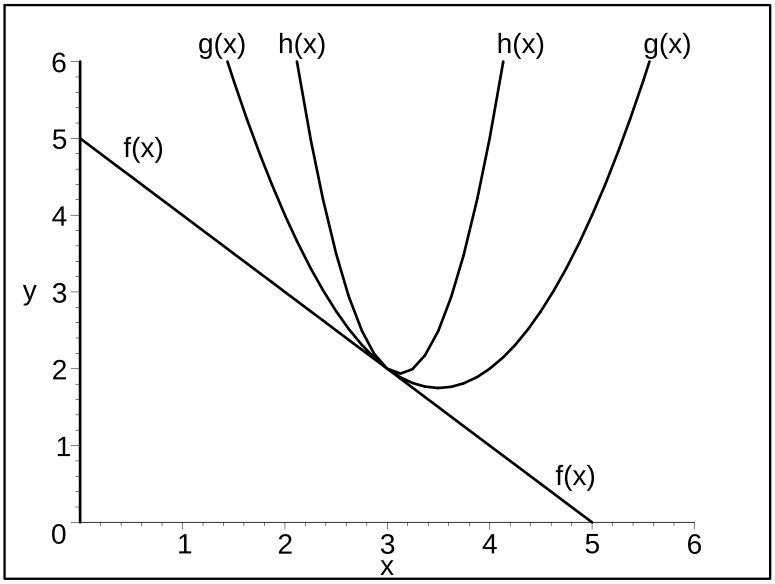
$$dy/dt = 3t^2(y-4).$$

For each equilibrium, tell whether it is stable or unstable.

6. (6 pts) Evaluate:

$$\int_{-1}^{2} \pi r e^{-3r}\, dr.$$

7. (9 pts) Three functions $f(x)$, $g(x)$, and $h(x)$ are graphed below. Propose possible Taylor series centered at $x = 3$ for these three functions. It is important that your formulas for the three be compatible with each other. Include enough terms to distinguish between the functions.



$f(x) =$

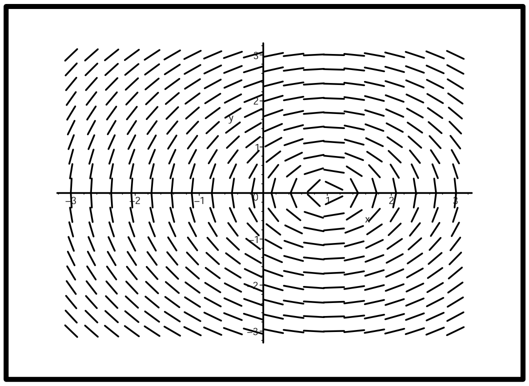
$g(x) =$

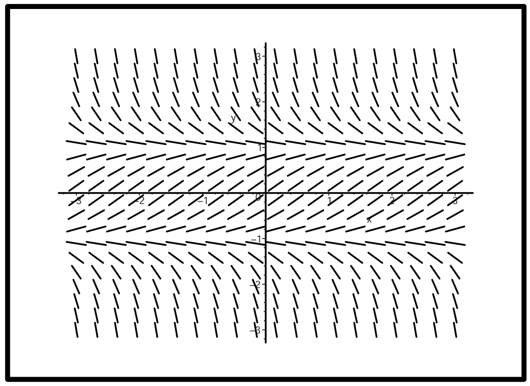
$h(x) =$

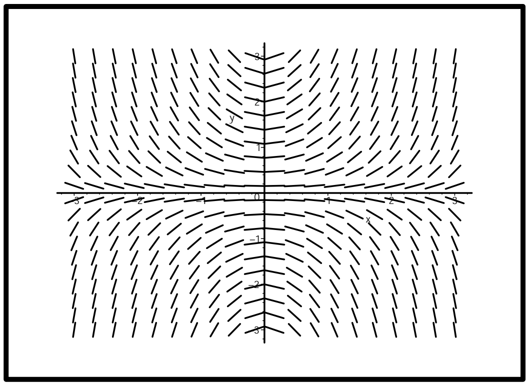
8. (Matching, 2 pts each) For each of the slope fields below, find its equation from the list of differential equations, and place the letter of its equation beside it.

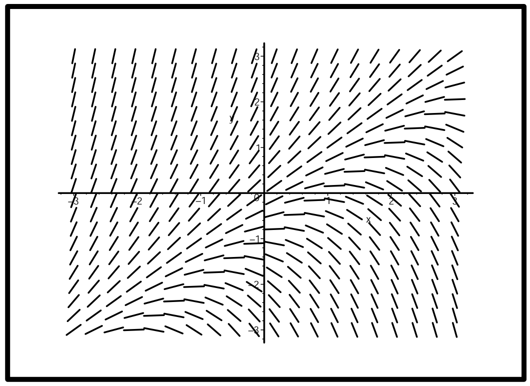
A: $\frac{dy}{dx} = y(1 - \frac{y}{2}) - 1$ C: $\frac{dy}{dx} = x - 1 - y$ E: $\frac{dy}{dx} = \frac{1-x}{y}$ G: $\frac{dy}{dx} = (x-1)^2 + y^2$

B: $\frac{dy}{dx} = x^3$ D: $\frac{dy}{dx} = 1 - x + y$ F: $\frac{dy}{dx} = xy$ H: $\frac{dy}{dx} = 1 - y^2$









9. (6 pts) The chambered nautilus builds a spiral sequence of closed chambers. It constructs them from the inside out, with each chamber approximately 20% larger (by volume) than the last. (The large open section at the top is not a "chamber.") The largest chamber is 9 cubic inches. How much volume is enclosed by all the chambers? Assume for simplicity that there are infinitely many chambers. Show your work.

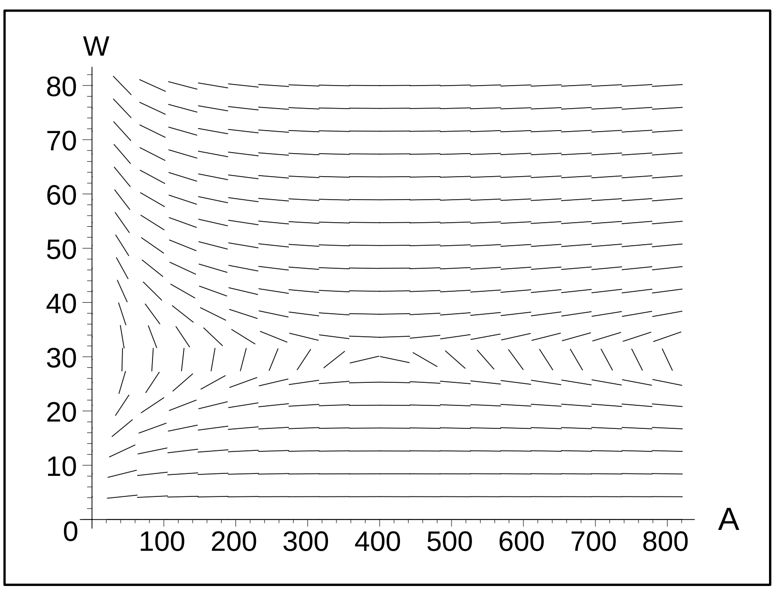
10. (11 pts) The Whistling Thorn Acacia tree and a certain species of stinging ant closely share the same environment. The trees harbor and feed the ants, which, in turn, protect the trees from mammals and other insects. Assume:

1. The yearly birth rate of the ant population $A(t)$ is proportional to the product of the number of ants and number of trees, with constant of proportionality 10^{-2}.
2. The **relative** yearly death rate of the ants is 30%.
3. The yearly birth rate of the tree population $W(t)$ is proportional to the product of the number of ants and the number of trees, with constant of proportionality 10^{-4}.
4. The **relative** yearly death rate of the trees is 4%.

a. (5 pts) Write a system of differential equations for these populations. Use variables A and W for the populations. No explanation necessary.

b. (1 pt) In your response to part a., circle the term which best expresses the truth "ants help trees." No explanation necessary.

c. (5 pts) Here is a picture of the phase plane for the system. At time $t = 0$, $A = 50$ and $W = 40$. Give an accurate graph of each $A(t)$ and $W(t)$. No explanation necessary. Use the back if you prefer.



11. (8 pts) The area surrounding a geyser is found to have high quantities of mineral deposits. Assume that at any radius $r < 5$ meters, $5 - r$ grams per square meter of the mineral silica can be found. How much silica is there, in total, within 5 meters of the geyser? Show your work.