

Math 116 — Second Midterm

March 19, 2012

Name: _____

Instructor: _____ Section: _____

1. **Do not open this exam until you are told to do so.**
2. This exam has 11 pages including this cover. There are 9 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones and pagers**, and remove all headphones.

Problem	Points	Score
1	10	
2	11	
3	10	
4	13	
5	13	
6	8	
7	9	
8	12	
9	14	
Total	100	

1. [10 points] Indicate if each of the following statements are true or false by circling the correct answer. **You do not need to justify your answers.**

a. [2 points] The integral $\int_{-2}^2 \frac{1}{x^2} dx = -1$

True False

b. [2 points] For any positive number p , the integral $\int_0^\infty \frac{1}{x^p} dx$ diverges.

True False

- c. [2 points] If the median grade of an exam is larger than the average grade then more than half of the students got a grade greater or equal to the average.

True False

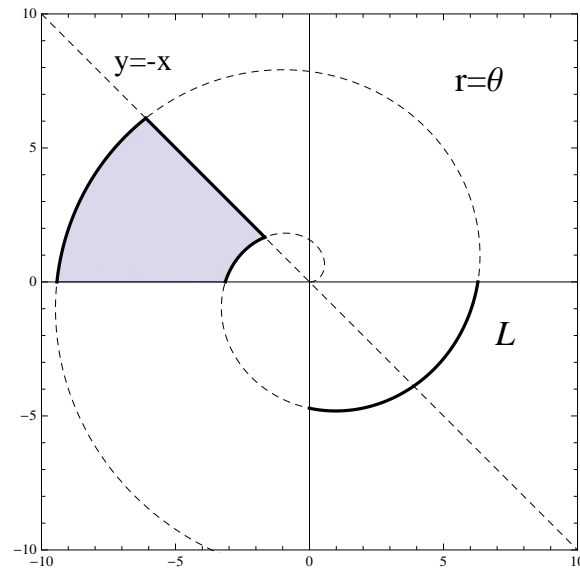
d. [2 points] Let $f(x)$ be a positive and continuous function. If $\lim_{x \rightarrow \infty} f(x) = \infty$, then $\int_0^\infty \frac{1}{f(x)} dx$ converges.

True False

- e. [2 points] The line $y = 2x + 1$ has parametric equations $x = -1 + 2t$, $y = -1 + 4t$.

True False

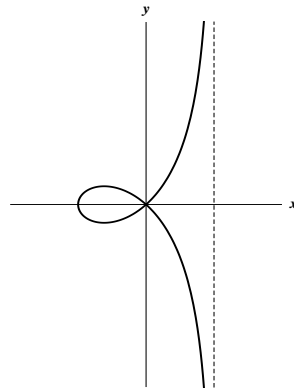
2. [11 points] Consider the graph of the spiral $r = \theta$ for $\theta \geq 0$.



In the following questions, write an expression (you do not need to evaluate any integrals) involving definite integrals that computes the values of the following quantities :

- a. [4 points] The length of the arc L .
- b. [7 points] The area of the shaded region.

3. [10 points] The motion of a particle is given by the following parametric equations



$$x(t) = \frac{a(t^2 - 1)}{t^2 + 1} \qquad y(t) = \frac{t^3 - t}{t^2 + 1}.$$

for $-\infty < t < \infty$ and $a > 0$. Show all your work to receive full credit.

- a. [3 points] Find the values of t at which the particle passes through the origin.
- b. [5 points] Find the value of t at which the curve defined by the parametric equations has a vertical tangent line. Also, give the (x, y) coordinates of this point.
- c. [2 points] The curve has a vertical asymptote. Find the equation of this asymptote.

4. [13 points]

- a. [6 points] A cylindrical tank with height 8 m and radius of 8 m is standing on one of its circular ends. The tank is initially empty. Water is added at a rate of $2 \text{ m}^3 / \text{min}$. A valve at the bottom of the tank releases water at a rate proportional to the water's depth (proportionality constant = k). Let $V(t)$ be the volume of the water in the tank at time t , and $h(t)$ be the depth of the water at time t .

i. Find a formula for $V(t)$ in terms of $h(t)$. $V(t) =$ _____

- ii. Find the differential equation satisfied by $V(t)$. Include the appropriate initial conditions.

Differential equation: _____ Initial condition: _____

- b. [7 points] Let $M(t)$ be the balance in dollars in a bank account t years after the initial deposit. The function $M(t)$ satisfies the differential equation

$$\frac{dM}{dt} = \frac{1}{100}M - a.$$

where a is a positive constant. Find a formula for $M(t)$ if the initial deposit is 1,000 dollars. Your answer may depend on a .

5. [13 points] Consider the following differential equations

A. $y' = 2x$

B. $y' = 5y - 1$

C. $yy' = 2$

D. $y' = \frac{y}{x}$

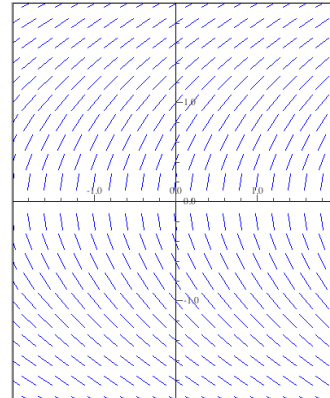
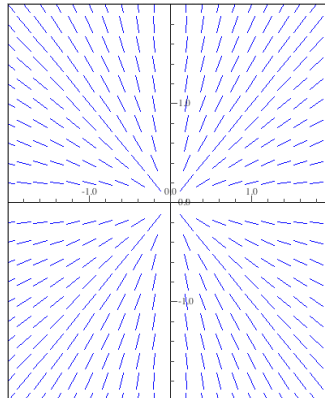
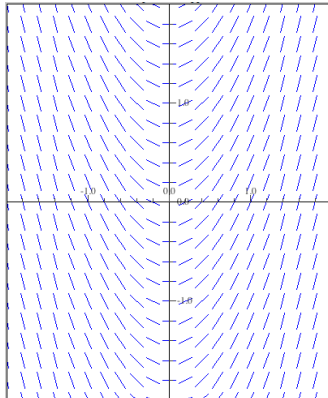
a. [6 points] Each of the following functions is a solution to one of the differential equations listed above. Indicate which differential equation with the corresponding letter (A,B,C or D) on the given line.

1. $y = \frac{1}{5} + e^{5x}$ _____

3. $y = 2\sqrt{x}$ _____

2. $y = x^2 + 1$ _____

b. [3 points] Each of the following slope fields belongs to one of the differential equations listed above. Indicate which differential equation on the given line.



c. [4 points] Find the equilibrium solutions of the differential equations given above (if any). Write the equation of the equilibrium solutions in the space provided below. If the equation does not have equilibrium solutions, write none.

A. _____

B. _____

C. _____

D. _____

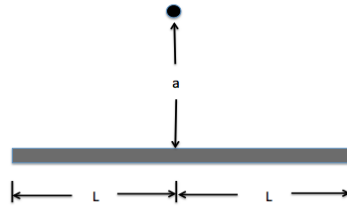
6. [8 points] A box is dropped from an airplane. The downward velocity $v(t)$ of the box, once its parachute opens, satisfies the differential equation

$$\frac{dv}{dt} = 10 - \frac{1}{10}(1 + e^{-t})v^2.$$

Suppose the parachute opens when the velocity of the box is 11 m/s. Use Euler's method with three steps to approximate the velocity of the box one second after the parachute opens. Fill in the table below with the approximations at each step. Be sure to include all your work to receive full credit.

t	0			
$v(t)$				

7. [9 points] If a particle of mass m is positioned at a perpendicular distance a from the center of a rod of length $2L$ and constant mass density δ as shown below



The force of gravitational attraction F between the rod and the particle is given by

$$F = Gm\delta a \int_{-L}^L \frac{1}{(a^2 + x^2)^{\frac{3}{2}}} dx.$$

- a. [5 points] Does the force of gravitational attraction F approach infinity as the length of the rod goes to infinity? Justify your answer using the comparison test.

- b. [4 points] Consider the integral

$$I = \int_1^{\infty} \frac{1}{(a^2 + x^2)^p} dx$$

- i. Give a power function that if integrated over $[1, \infty)$ will have the same convergence or divergence behavior as I .

- ii. For what values of p is I convergent or divergent?

8. [12 points] Determine if the following integrals converge or diverge. Justify your answer. If you use the comparison test, be sure to show all your work.

a. [3 points] $\int_1^{\infty} \frac{1}{x + e^x} dx.$

b. [4 points] $\int_1^e \frac{1}{x(\ln x)^2} dx.$

c. [5 points] $\int_{2\pi}^{\infty} \frac{x \cos^2 x + 1}{x^3} dx.$

9. [14 points] A machine produces copper wire, and occasionally there is a flaw at some point along the wire. The length x of wire produced between two consecutive flaws is a continuous variable with probability density function

$$f(x) = \begin{cases} c(1+x)^{-3} & \text{for } x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Show all your work in order to receive full credit.

- a. [5 points] Find the value of c .

- b. [2 points] Find the cumulative distribution function $P(x)$ of the density function $f(x)$. Be sure to indicate the value of $P(x)$ for **all** values of x .

c. [6 points] Find the mean length of wire between two consecutive flaws.

d. [1 point] A second machine produces the same type of wire, but with a different probability density function (pdf). Which of the following graphs could be the graph of the pdf for the second machine? Circle all your answers.

