On my honor, as a student,
I have neither given nor received
unauthorized aid on this academic work. **Initials:** ______

Do not write in this area

Math 116 — First Midterm — February 5, 2018

Your Initials Only: ______ Your U-M ID # (not uniqname): ____________________

Instructor Name: ______________________________ Section #: __________

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 11 pages including this cover. Do not separate the pages of this exam. If pages do become separated, write your UMID on every page and point this out to your instructor when you hand in the exam.
4. There are 10 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
5. Note that the back of every page of the exam is blank, and, if needed, you may use this space for scratchwork. Clearly identify any of this work that you would like to have graded.
6. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
7. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it.
8. The use of any networked device while working on this exam is not permitted.
9. You may use any one calculator that does not have an internet or data connection except a TI-92 (or other calculator with a "qwerty" keypad). However, you must show work for any calculation which we have learned how to do in this course.
   You are also allowed two sides of a single $3'' \times 5''$ notecard.
10. For any graph or table that you use to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how you used the graph or table to find the answer.
11. Include units in your answer where that is appropriate.
12. Problems may ask for answers in *exact form.* Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is not.
13. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones, earbuds, and smartwatches. Put all of these items away.
14. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	14	
2	9	
3	10	
4	10	
5	9	

Problem	Points	Score
6	7	
7	10	
8	14	
9	9	
10	8	
Total	100	

1. [14 points] Let $f(x)$ be a twice-differentiable function. Use the table to compute the following expressions. Show your work.

x	0	1	2	3	4	5	6	7	8	9
$f(x)$	1	2	4	11	1	3	5	4	2	3
$f'(x)$	2	3	7	4	-5	2	1	-2	-3	1

a. [3 points] $\displaystyle\int_1^8 \frac{f'(\sqrt[3]{x})}{\sqrt[3]{x^2}}\,dx$

Answer: a. ____________________

b. [3 points] $\displaystyle\int_7^9 \frac{12f'(x)}{(f(x))^2}\,dx$

Answer: b. ____________________

c. [3 points] $\displaystyle\int_0^3 xf''(x)\,dx$

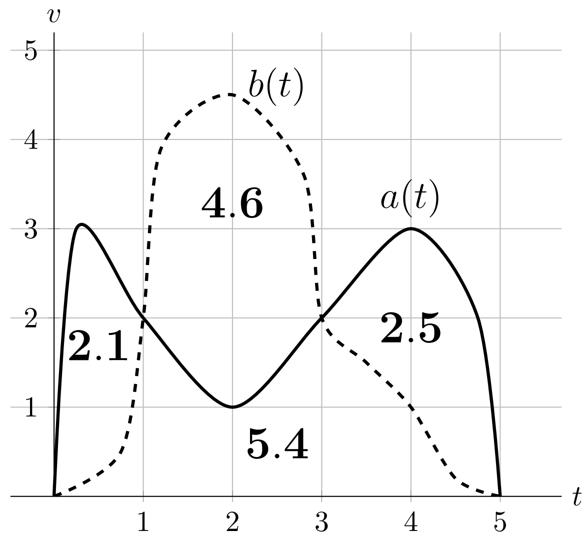
Answer: c. ____________________

d. [5 points] The average value of $\dfrac{2f'(x)}{(f(x))^2+f(x)}$ on $[4,6]$.

Answer: d. ____________________

2. [9 points]

When Alejandra and Brontel were children they spent summer mornings chasing birds in flight. One memorable day they encountered an owl. The following graph shows the velocities $a(t)$ of Alejandra (solid) and $b(t)$ of Brontel (dashed), measured in meters per second, t seconds after the owl took off. The area of each region is given.



a. [1 point] How far (in meters) do Alejandra and Brontel chase the owl?

Answer: ____________________

b. [5 points] Suppose the owl ascends to a height of h meters according to $h(t) = \sqrt{t}$ where t is seconds since it went airborne. Let $L(h)$ be the number of meters Alejandra is ahead of Brontel when the owl is h meters above ground. Write an expression for $L(h)$ involving integrals and compute $L'(2)$.

Answer: $L(h) =$ ____________________

$L'(2) =$ ____________________

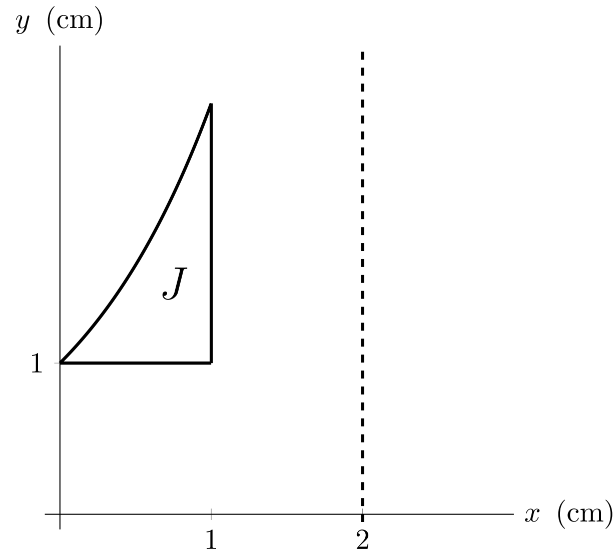
c. [3 points] The next bird to pass is a dove. This time Alejandra runs twice as fast and Brontel runs three times as fast as they did when chasing the owl. How much faster (in m/s) is Brontel than Alejandra on average in the first 5 seconds?

Answer: ____________________

3. [10 points]

Debra McQueath hooked you up with an interview at `Print.juice`. Being a legitimate tech start-up, the `Print.juice` interview consists of answering technical questions on the spot. Debra gave you the following questions for practice.

The region J is a common `Print.juice` shape. It is bounded by $x = 1$, $y = 1$, and $y = e^x$.



a. [3 points] First, consider the solid with base J and square cross sections perpendicular to the x-axis. If the density of the solid is a function of the x-coordinate $a(x)$ g/cm^3, write an integral that represents the total mass of the solid in grams.

Answer: ______________________________

For b. and c., consider the solid made by rotating J around the line $x = 2$.

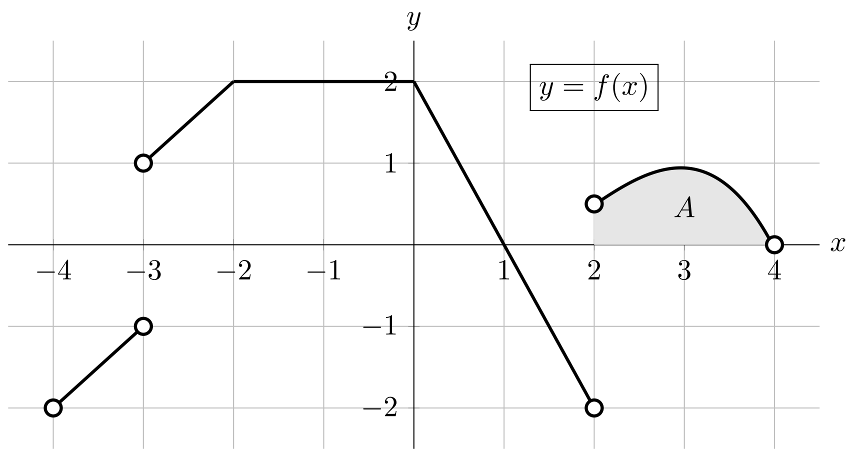
b. [3 points] If the density of the solid is a function of the y-coordinate $b(y)$ g/cm^3, write an integral that represents the total mass of the solid in grams.

Answer: ______________________________

c. [4 points] If the density of the solid is a function of the distance r cm from the axis of rotation $c(r)$ g/cm^3, write an integral that represents the total mass of the solid in grams.

Answer: ______________________________

4. [10 points] The entire graph of the function $f(x)$ is given below. Note that $f(x)$ is piecewise linear on $(-4, 2)$, and the area of the shaded region A is 1.5.



a. [2 points] Let $F(x)$ be the continuous antiderivative of $f(x)$ passing through $(2, 1)$. Circle all of the x-coordinates listed below at which $F(x)$ appears to have an inflection point.

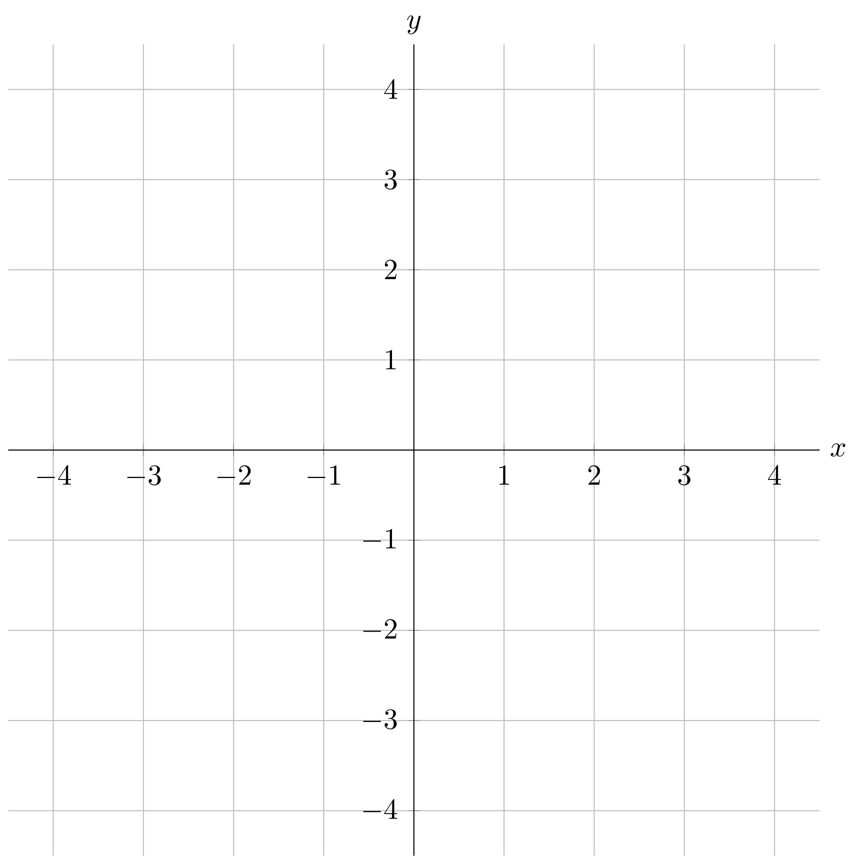
$x = -3$

$x = 1$

$x = 2$

$x = 3$

NONE OF THESE



b. [8 points] On the axes to the right, sketch a graph of the function $G(x)$, where $G(x)$ is a continuous antiderivative of $f(x)$ on $(-4, 4)$ and on the interval $(-3, 2)$, $G(x)$ is given by

$$G(x) = \int_{-1}^{x} f(t)\, dt.$$

Make sure that local extrema and concavity are clear. If there are features that are difficult for you to draw, indicate these on your graph.

5. [9 points] Tammy Toppel is directing a performance art piece at the community center. She fills a large cone with sand and cuts a small hole in the bottom. Gerd Hömf was hired from a temp agency to stand behind the scenes and steadily lift the cone with an elaborate pulley system, letting the sand slowly spill onto the stage.

a. [2 points] The filled cone starts with a total mass of 40 kilograms and spills sand at a constant rate of 1/2 a kilogram per second once it is lifted. Tammy wants Gerd to lift the cone at a constant rate of r meters per second. Find a formula for the mass $M(h)$, in kilograms, of the cone when it is h meters above the stage.

Answer: $M(h) =$ ______________________

b. [4 points] Gerd lifts the cone until it reaches a height of 20 meters above the stage. Write an integral which represents the work (measured in Joules) done by Gerd while lifting the cone. The integral may include the rate r at which Gerd lifts and g the acceleration (in $\mathrm{m/s^2}$) due to gravity.

Answer: ______________________

c. [3 points] There's one catch: Gerd's contract strictly prohibits him from exerting more than $500g$ Joules of work, where g is the acceleration due to gravity. At what rate r (in m/s) should Tammy ask Gerd to lift in order to not violate his contract and to get the cone lifted as quickly as possible?

Answer: $r =$ ______________________

6. [7 points] For each of the questions below, circle **all** of the available correct answers. You must circle at least one choice to receive any credit. No credit will be awarded for unclear markings. No justification is necessary.

a. [3 points] Which $F(x)$ are antiderivatives of $f(x) = e^{x^2}$ for $x > 0$ with $F(3) = 5$?

I. $F(x) = \displaystyle\int_0^{x^2} e^u \, du + 5$

II. $F(x) = \displaystyle\int_3^{x} 5e^{u^2} \, du$

III. $F(x) = \dfrac{1}{x^2} e^{x^2} + 5$

IV. $F(x) = \displaystyle\int_{x^2}^{9} -\frac{1}{2\sqrt{u}} e^u \, du + 5$

V. $F(x) = \displaystyle\int_3^{x} e^{u^2} \, du + 5$

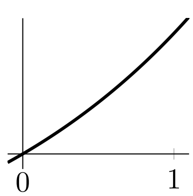
VI. $F(x) = \dfrac{e^{x^2}}{2x} - \dfrac{e^9}{6} + 5$

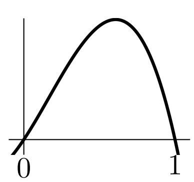
b. [2 points] Suppose $f(x)$ is an odd function. Which values of b make the following equation true?

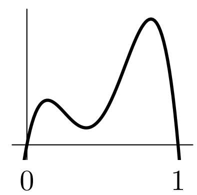
$$\int_{-\pi}^{b} \sin(f(x)) \, dx = 0$$

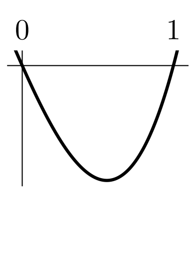
I. $b = -\pi$ II. $b = 0$ III. $b = \pi$ IV. $b = \frac{3\pi}{2}$ V. $b = 2\pi$

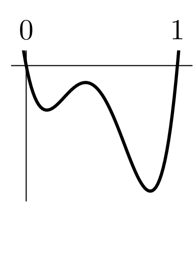
c. [2 points] Which of the following could be the graph of $f(x) = \displaystyle\int_x^{x^3} e^{\sqrt[3]{u}} \, du$?

I.


II.


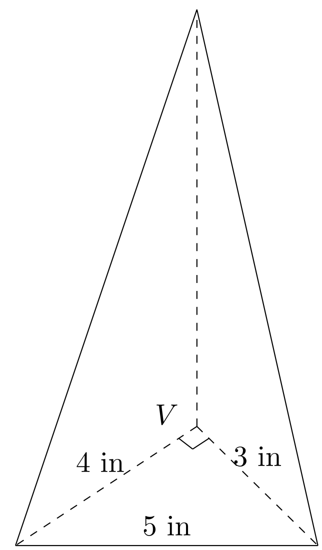
III.


IV.


V.


7. [10 points]

Ms. Parth made a pyramid for her niece and nephew. The pyramid is 10 inches tall and the base has the shape of a right triangle. When the pyramid is sitting on the table it looks like the figure to the right. The three angles at vertex V are right angles. (Dashed lines are not visible from this point of view. The figure may not be drawn to scale.)



a. [6 points] Write an integral that represents the total volume of the pyramid in cubic inches and evaluate it.

Answer: Integral: ____________________

Volume: ____________

b. [4 points] The children fail to share the pyramid, so Ms. Parth decides to cut it parallel to the table into two pieces of equal volume. How many inches H from the **top** of the pyramid should Ms. Parth cut? Round your answer to the nearest tenth of an inch.

Answer: $H =$ ________________

8. [14 points] Let $g(x)$ be a differentiable function with domain $(-1, 10)$ where some values of $g(x)$ and $g'(x)$ are given in the table below. Assume that all local extrema and critical points of $g(x)$ occur at points given in the table.

x	0	1	2	3	4	5	6	7	8
$g(x)$	2.0	3.3	5.7	6.8	6.0	4.3	2.4	0.2	-4.9
$g'(x)$	2.8	2.5	2.0	0.0	-1.4	-1.9	-1.6	-3.0	-8.1

a. [3 points] Estimate $\int_0^8 g(x)\,dx$ using RIGHT(4). Write out each term in your sum.

b. [4 points] Approximate the area of the region between $g(x)$ and the function $f(x) = x + 2$ for $0 \le x \le 4$, using MID(n) to estimate any integrals you use. Use the greatest number of subintervals possible, and write out each term in your sums.

c. [3 points] Is your answer to **b.** an overestimate, an underestimate, or is there not enough information to tell? Briefly justify your answer.

Answer: (circle one)

OVERESTIMATE UNDERESTIMATE NOT ENOUGH INFORMATION

d. [4 points] Write an integral giving the arc length of $y = g(x)$ between $x = 2$ and $x = 8$. Estimate this integral using TRAP(2). Write out each term in your sum.

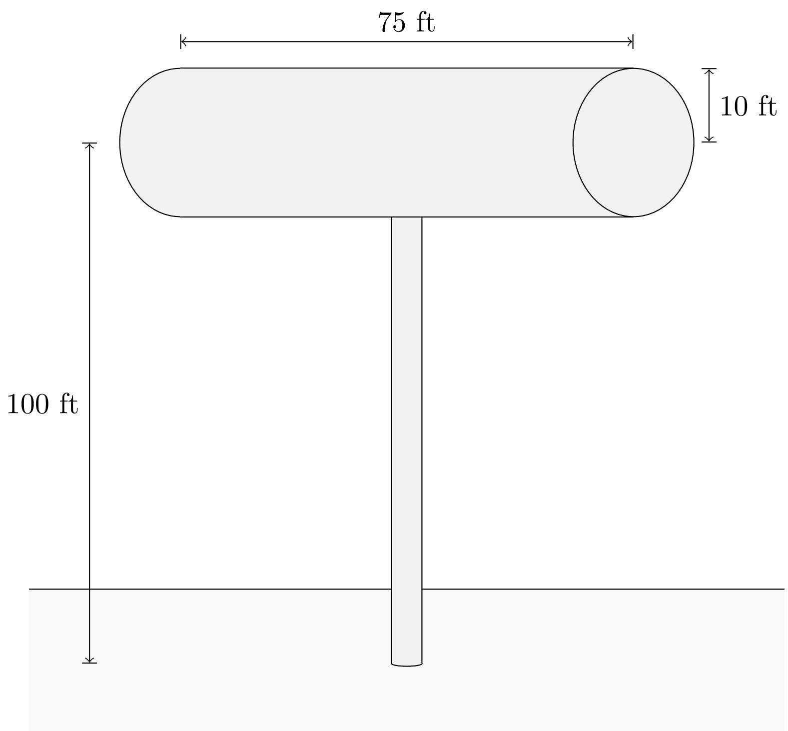
Answer: Integral: ____________________

Answer: TRAP(2)= ____________________

9. [9 points]

De'von Baptiste is a shrewd industrialist. When energy costs are low, De'von pumps purified muck (which he gets *for free* from the city) into very tall tanks. In this way he stores cheap potential energy. Someday, when energy prices soar, Mr. Batiste will convert it all back into useful kinetic energy at a great profit.

His tanks are cylinders 75 ft long with radius 10 ft. The center of a tank is 100 ft above the ground. Purified muck has a density of 800 pounds/ft^3.



a. [3 points] What is the area, in square feet, of a cross-section parallel to the ground taken y feet above the **center** of the tank?

Answer: ____________________

b. [6 points] Write an integral which represents the total work (in foot-pounds) required to fill one of De'von Batiste's tanks with purified muck. **Do not evaluate this integral.**

Answer: ____________________

10. [8 points] For each of the questions below, circle all of the available correct answers. Circle NONE OF THESE if none of the available choices are correct. You must circle at least one choice to receive any credit. No credit will be awarded for unclear markings. No justification is necessary.

a. [3 points] Which of the following integrals are equal to $\displaystyle\lim_{n\to\infty}\sum_{k=0}^{n-1}\left(3+\frac{2k}{n}\right)^4\cdot\frac{2}{n}$?

i. $\displaystyle\int (1+kx)^4 x\,dx$

ii. $\displaystyle\int_3^5 x^4\,dx$

iii. $\displaystyle\int_0^{n-1}\left(3+\frac{2x}{n}\right)^4\cdot\frac{2}{n}\,dx$

iv. $\displaystyle\int_2^3 x^4\,dx$

v. $\displaystyle\int_0^2 (3+x)^4\,dx$

vi. NONE OF THESE

b. [3 points] Which of the following expressions give the volume of the solid made by rotating around the y-axis the region bounded by $y=\sin(x)$, $y=0$, and $x=\frac{\pi}{2}$?

i. $\displaystyle\int_0^{\pi/2}\pi\left(\frac{\pi}{2}-\sin(x)\right)^2\,dx$

ii. $\displaystyle\int_0^{\pi/2}\pi\left(\left(\frac{\pi}{2}\right)^2-\sin^2(x)\right)\,dx$

iii. $\displaystyle\int_0^{\pi/2} 2\pi x\sin(x)\,dx$

iv. $\displaystyle\int_0^{\pi/2}\pi\sin^2(x)\,dx$

v. $\displaystyle\int_0^1\pi\left(\frac{\pi}{2}-\arcsin(y)\right)^2\,dy$

vi. $\displaystyle\int_0^1\pi\left(\left(\frac{\pi}{2}\right)^2-(\arcsin(y))^2\right)\,dy$

vii. $\displaystyle\int_0^1 2\pi y\arcsin(y)\,dy$

viii. NONE OF THESE

c. [2 points] Let $f(x)$ be a function that is increasing on $(-3,3)$, concave up on $(0,3)$, and has a point of inflection at $x=0$. Consider the approximations for $\displaystyle\int_{-2}^2 f(x)\,dx$ given by LEFT(n) and TRAP(n). Which of the following statements **must** be true?

i. $\text{TRAP}(n) < \displaystyle\int_{-2}^2 f(x)\,dx$

ii. $\text{TRAP}(n) > \displaystyle\int_{-2}^2 f(x)\,dx$

iii. TRAP(n) is neither an overestimate nor an underestimate for $\displaystyle\int_{-2}^2 f(x)\,dx$.

iv. $\text{LEFT}(n) < \displaystyle\int_{-2}^2 f(x)\,dx$

v. $\text{LEFT}(n) > \displaystyle\int_{-2}^2 f(x)\,dx$

vi. $\text{LEFT}(n) = 0$

vii. NONE OF THESE