

Do not write in this area

On my honor, as a student,
I have neither given nor received
unauthorized aid on this academic work. **Initials:** _____

Math 116 — Final Exam — April 19, 2018

Your Initials Only: _____ Your U-M ID # (not unickname): _____

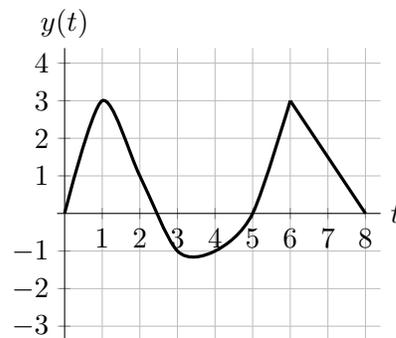
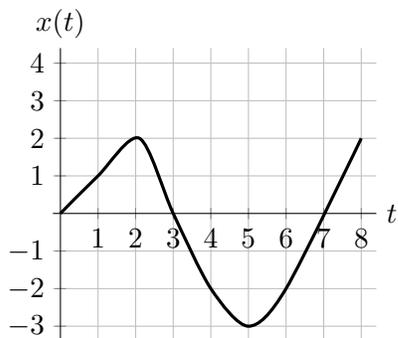
Instructor Name: _____ Section #: _____

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 12 pages including this cover. Do not separate the pages of this exam. If pages do become separated, write your UMID on every page and point this out to your instructor when you hand in the exam.
4. There are 12 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
5. Note that the back of every page of the exam is blank, and, if needed, you may use this space for scratchwork. Clearly identify any of this work that you would like to have graded.
6. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
7. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it.
8. The use of any networked device while working on this exam is not permitted.
9. You may use any one calculator that does not have an internet or data connection except a TI-92 (or other calculator with a “qwerty” keypad). However, you must show work for any calculation which we have learned how to do in this course.
You are also allowed two sides of a single 3" × 5" notecard.
10. For any graph or table that you use to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how you used the graph or table to find the answer.
11. Include units in your answer where that is appropriate.
12. Problems may ask for answers in *exact form*. Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is not.
13. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones, earbuds, and smartwatches. Put all of these items away.
14. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	8	
2	9	
3	10	
4	11	
5	10	
6	11	

Problem	Points	Score
7	6	
8	8	
9	7	
10	8	
11	6	
12	6	
Total	100	

1. [8 points] Once again, Giuseppe Li has a nightmare. This time he is roaming the plains in a covered wagon. His stepfather, Gerd Hömf, plotted the x - and y -coordinates of Giuseppe's dream wagon measured in astral miles as a function of dream days t . The positive y -direction is north, and the positive x -direction is east.



- a. [2 points] At what time(s) t was Giuseppe's wagon moving directly north? Directly south? If there are no such times, write "NONE".

Answer: Directly north at $t =$ _____

Answer: Directly south at $t =$ _____

- b. [2 points] How far, in astral miles, from where it began was Giuseppe's wagon after 8 dream days?

Answer: _____

- c. [2 points] What was Giuseppe's approximate speed, in astral miles per dream days, at time $t = 7$?

Answer: _____

- d. [2 points] Approximate the total distance, in astral miles, that Giuseppe's wagon traveled during the first 2 dream days.

Answer: _____

2. [9 points] For a class project, Yennifer is studying the accumulation of dead leaves on the ground in a particular region on Nichols Arboretum.
- a. [4 points] She finds that the dead leaves accumulate at a constant rate of 6 grams per square centimeter per year. At the same time, the leaves on the ground decompose at a continuous rate of 80 percent per year. Write a differential equation for the total quantity Q of dead leaves, in grams per square centimeter, at time t , in years.

Answer: _____

- b. [5 points] Yennifer finds that if she covers the ground in purified muck, then the total quantity P , in grams, of dead leaves per square centimeter satisfies the differential equation

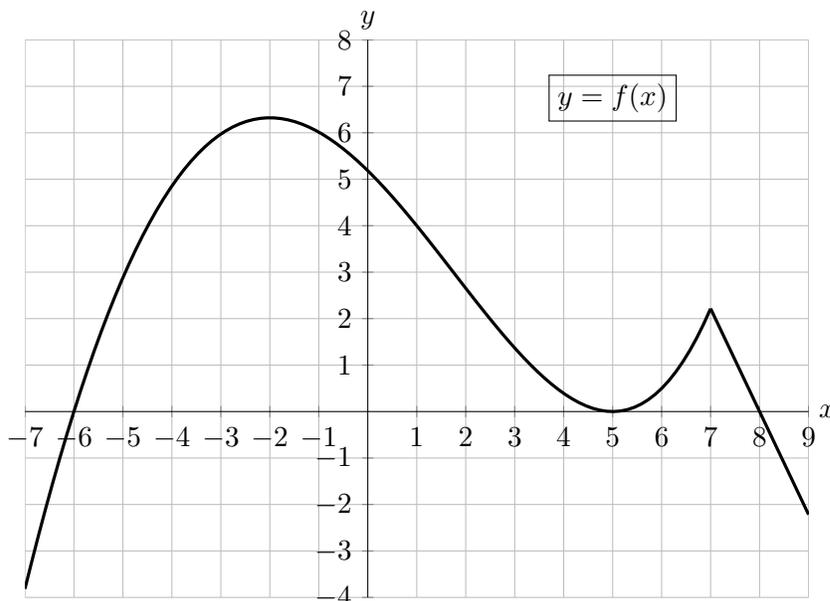
$$\frac{dP}{dt} = (P - 3) \cos(2\pi t)$$

In addition, when she first applies the muck (at $t = 0$), the ground is covered with 1 gram per square centimeter of leaves.

Use separation of variables to find a formula for $P(t)$. Show your work.

Answer: $P(t) =$ _____

3. [10 points] Consider the function $f(x)$ graphed below.



- a. [3 points] Let $F(x) = \int_0^x f(t) dt$. Find the x -coordinates of all local extrema of $F(x)$ and classify them as local maxima or local minima. Write “NONE” if there are none.

Answer: Local maxima at $x =$ _____

Answer: Local minima at $x =$ _____

- b. [3 points] Let $G(x) = \int_{3x}^{x^2} f(t) dt$. Compute $G'(-1)$.

Answer: $G'(-1) =$ _____

- c. [2 points] Which approximation method is guaranteed to underestimate $\int_{-4}^0 f(x) dx$?

MID TRAP LEFT RIGHT NONE OF THESE

- d. [2 points] Which approximation method is guaranteed to overestimate $\int_{-1}^5 f(x) dx$?

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4. [11 points]

- a. [6 points] Determine whether the following series converges absolutely, converges conditionally, or diverges, and give a complete argument justifying your answer.

$$\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{1}{n}\right)$$

Converges absolutely

Converges conditionally

Diverges

Justification:

- b. [5 points] Compute the value of the following improper integral. **Show all your work using correct notation.** Evaluation of integrals must be done **without a calculator.**

$$\int_0^{\infty} \frac{e^x}{(1+e^x)^2} dx$$

5. [10 points] Compute the **exact value** of each of the following. You do not need to show work.

a. [2 points] Find the radius of convergence R of $\sum_{n=1}^{\infty} \frac{5(x-1)^n}{3^n}$.

Answer: $R =$ _____

b. [2 points] $\sum_{n=1}^{100} e^n$

Answer: $\sum_{n=1}^{100} e^n =$ _____

c. [2 points] $\lim_{n \rightarrow \infty} \int_{-\infty}^n p(t) dt$, where $p(t)$ is a probability density function.

Answer: $\lim_{n \rightarrow \infty} \int_{-\infty}^n p(t) dt =$ _____

d. [2 points] Find the function $f(x)$ satisfying $\int x^3 e^x dx = x^3 e^x - \int f(x) dx$

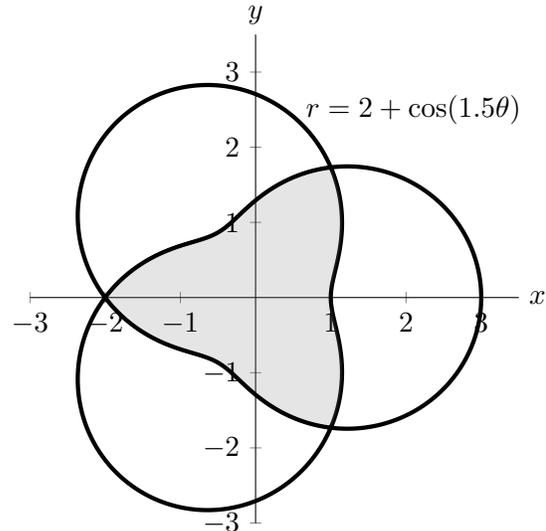
Answer: $f(x) =$ _____

e. [2 points] $\sum_{n=0}^{\infty} \frac{(-4)^n}{(2n)!}$

Answer: $\sum_{n=0}^{\infty} \frac{(-4)^n}{(2n)!} =$ _____

6. [11 points]

Leight Vloss has instructed his Star Children to run laps on the Trail of Atonement. The Trail of Atonement is best described as the polar curve $r = 2 + \cos(1.5\theta)$ where r is measured in kilometers. An aerial view of the trail is illustrated below.



- a. [4 points] Leight stands on a pedestal in the center of the trail (at the origin). What is the furthest distance, in km, a Star Child gets from Leight on the Trail of Atonement? List all angles θ in $[0, 4\pi)$ where this distance r is achieved.

Answer: Greatest distance: _____

Answer: $\theta =$ _____

- b. [3 points] Write an integral in terms of θ which represents the total length, in km, of the Trail of Atonement.

- c. [4 points] The shaded innermost region of the trail is called the Sacred Heart. Write an expression involving one or more integrals which represents the area, in km^2 , of the Sacred Heart.

7. [6 points] The function $r(t)$, defined for all real numbers t , gives the position of a particle moving along the unit circle, where

$$r(t) = (\cos(t - t^3), \sin(t - t^3)).$$

- a. [3 points] Find all values of t where the particle stops moving.

Answer: $t =$ _____

- b. [3 points] For which values of t is the particle moving counterclockwise?

Answer: _____

8. [8 points] Let $f(x) = xe^{-x^2}$.

- a. [4 points] Find the first four nonzero terms of the Taylor series for $f(x)$ centered at $x = 0$.

Answer: _____

- b. [2 points] Find the value of $f^{(18)}(0)$.

Answer: $f^{(18)}(0) =$ _____

- c. [2 points] Compute the limit

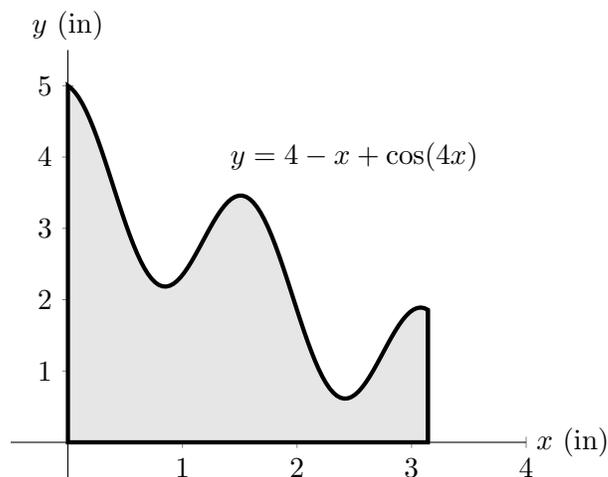
$$\lim_{x \rightarrow 0} \frac{xe^{-x^2} - x}{5x^3}.$$

Answer: $\lim_{x \rightarrow 0} \frac{xe^{-x^2} - x}{5x^3} =$ _____

10. [8 points] Recently Debra McQueath was thinking about all the great things she used to make at **Print.juice** by revolving regions around the y -axis. Those were the good days, weren't they?

a. [4 points]

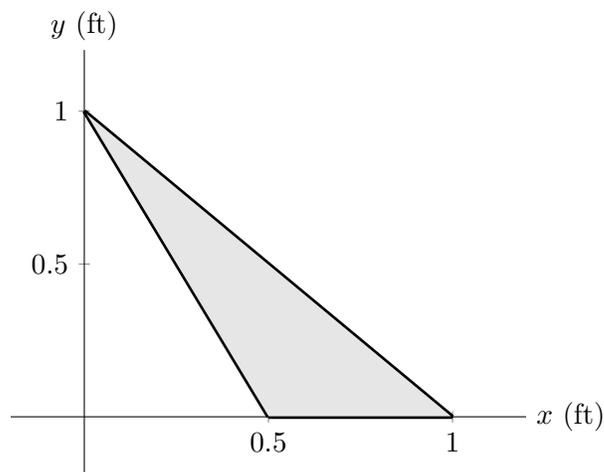
There was that one time she designed the Juice Titan™ formed by rotating the region in the first quadrant bounded by $x = \pi$ and $y = 4 - x + \cos(4x)$ around the y -axis. The density $\delta(x)$ of the plastic was a function of the distance from the center of the juicer, although Debra cannot quite remember what it was. Help Debra write an integral that represents the total mass of the Juice Titan™. Your integral may include the density function $\delta(x)$.



Answer: _____

b. [4 points]

On Debra's last day at **Print.juice** her team made her a commemorative hat containing a hollow chamber filled with juice by rotating the region bounded by $y = 0$, $y = 1 - x$ and $y = 1 - 2x$ around the y -axis. The juice-filled hat still sits on her kitchen table; she sometimes wonders how much juice is in the hat. Write an integral that represents the total volume of juice in the hat. **Note:** juice fills the solid formed by rotating the shaded region.



Answer: _____

11. [6 points] The polynomial $P_3(x) = 2 - 3(x + e)^2 + 5(x + e)^3$ is the third-degree Taylor polynomial approximating the function $g(x)$ for x near $-e$. Find the following values. Write "NI" if there is not enough information.

$g'(-e) =$ _____ $g(-e) =$ _____ $g'''(-e) =$ _____

$P_3^{(4)}(-e) =$ _____ $g(0) =$ _____ $P_3(0) =$ _____

12. [6 points] Match the differential equations to their corresponding slope fields.

i. $y' = x^2 + y^2$ _____

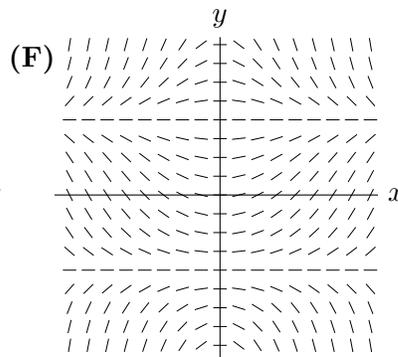
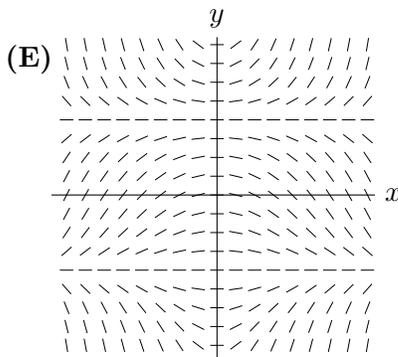
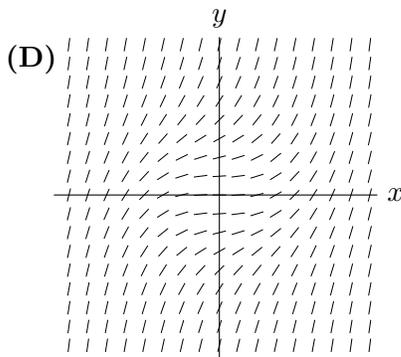
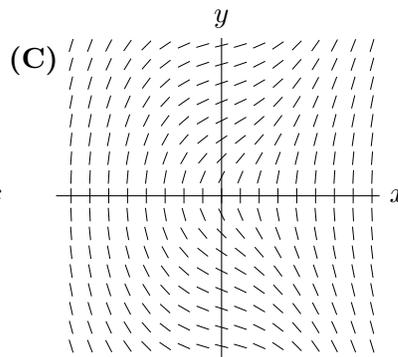
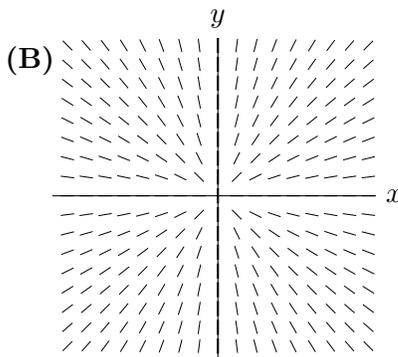
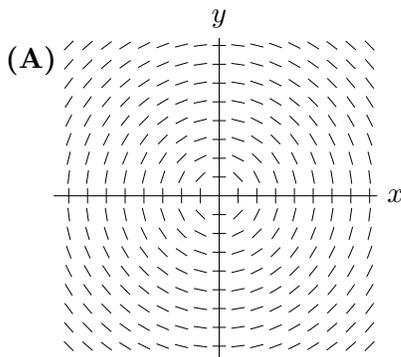
iv. $y' = x(y^2 - 1)$ _____

ii. $y' = \frac{y}{x}$ _____

v. $y' = x(1 - y^2)$ _____

iii. $y' = -\frac{x}{y}$ _____

vi. $y' = \frac{3x^2 + 1}{2y}$ _____



“Known” Taylor series (all around $x = 0$):

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \cdots \quad \text{for all values of } x$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \cdots + \frac{(-1)^n x^{2n}}{(2n)!} + \cdots \quad \text{for all values of } x$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots \quad \text{for all values of } x$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + \frac{(-1)^{n+1} x^n}{n} + \cdots \quad \text{for } -1 < x \leq 1$$

$$(1+x)^p = 1 + px + \frac{p(p-1)}{2!} x^2 + \frac{p(p-1)(p-2)}{3!} x^3 + \cdots \quad \text{for } -1 < x < 1$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots \quad \text{for } -1 < x < 1$$