Math 116 — First Midterm — February 5, 2018

EXAM SOLUTIONS

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 13 pages including this cover. Do not separate the pages of this exam. If pages do become separated, write your UMID on every page and point this out to your instructor when you hand in the exam.
4. There are 10 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
5. Note that the back of every page of the exam is blank, and, if needed, you may use this space for scratchwork. Clearly identify any of this work that you would like to have graded.
6. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
7. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it.
8. The use of any networked device while working on this exam is not permitted.
9. You may use any one calculator that does not have an internet or data connection except a TI-92 (or other calculator with a "qwerty" keypad). However, you must show work for any calculation which we have learned how to do in this course.
   You are also allowed two sides of a single $3'' \times 5''$ notecard.
10. For any graph or table that you use to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how you used the graph or table to find the answer.
11. Include units in your answer where that is appropriate.
12. Problems may ask for answers in *exact form.* Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is not.
13. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones, earbuds, and smartwatches. Put all of these items away.
14. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	14	
2	9	
3	10	
4	10	
5	9	

Problem	Points	Score
6	7	
7	10	
8	14	
9	9	
10	8	
Total	100	

1. [14 points] Let $f(x)$ be a twice-differentiable function. Use the table to compute the following expressions. Show your work.

x	0	1	2	3	4	5	6	7	8	9
$f(x)$	1	2	4	11	1	3	5	4	2	3
$f'(x)$	2	3	7	4	-5	2	1	-2	-3	1

a. [3 points] $\displaystyle\int_1^8 \frac{f'(\sqrt[3]{x})}{\sqrt[3]{x^2}}\,dx$

Solution: Use u-substitution with $u = \sqrt[3]{x}$ to get

$$u = \sqrt[3]{x}$$
$$du = \frac{1}{3}\frac{1}{\sqrt[3]{x^2}}dx.$$

Then the antiderivative is

$$\int \frac{f'(\sqrt[3]{x})}{\sqrt[3]{x^2}}\,dx = 3\int f'(u)\,du = 3f(\sqrt[3]{x}).$$

Hence we have

$$\int_1^8 \frac{f'(\sqrt[3]{x})}{\sqrt[3]{x^2}}\,dx = 3(f(2) - f(1)) = 6.$$

Answer: a. **6**

b. [3 points] $\displaystyle\int_7^9 \frac{12f'(x)}{(f(x))^2}\,dx$

Solution: Use u-substitution with $u = f(x)$ to get

$$u = f(x)$$
$$du = f'(x)dx.$$

Then the antiderivative is

$$\int \frac{12f'(x)}{(f(x))^2}\,dx = 12\int \frac{1}{u^2}\,du = -\frac{12}{f(x)}.$$

Hence we have

$$\int_7^9 \frac{12f'(x)}{(f(x))^2}\,dx = -\frac{12}{f(9)} + \frac{12}{f(7)} = -\frac{12}{3} + \frac{12}{4} = -1.$$

Answer: b. -1

c. [3 points] $\int_0^3 x f''(x)\, dx$

Solution: Use integration by parts with $u = x$ and $dv = f''(x)dx$ to get

$$\begin{aligned} u &= x & v &= f'(x) \\ du &= dx & dv &= f''(x)dx. \end{aligned}$$

Then the antiderivative is

$$\int x f''(x)\, dx = x f'(x) - \int f'(x)\, dx = x f'(x) - f(x).$$

$$\int_0^3 x f''(x)\, dx = (3f'(3) - f(3)) - (0f'(0) - f(0)) = (3 \cdot 4 - 11) - (0 \cdot 2 - 1) = 2.$$

Answer: c. **2**

d. [5 points] The average value of $\dfrac{2f'(x)}{(f(x))^2 + f(x)}$ on $[4, 6]$.

Solution: By definition, this average is given by the integral

$$\frac{1}{6-4}\int_4^6 \frac{2f'(x)}{(f(x))^2 + f(x)}\, dx = \int_4^6 \frac{f'(x)}{(f(x))^2 + f(x)}\, dx.$$

To compute this antiderivative, first use the u-substitution $u = f(x)$ to get

$$\int \frac{f'(x)}{(f(x))^2 + f(x)}\, dx = \int \frac{1}{u^2 + u}\, du.$$

Using the method of partial fractions we find the identity

$$\frac{1}{u^2 + u} = \frac{1}{u} - \frac{1}{u+1}.$$

Thus

$$\int \frac{1}{u^2 + u}\, du = \ln|u| - \ln|u+1|,$$

and changing variables back to x leads to

$$\int \frac{f'(x)}{(f(x))^2 + f(x)}\, dx = \ln|f(x)| - \ln|f(x)+1|.$$

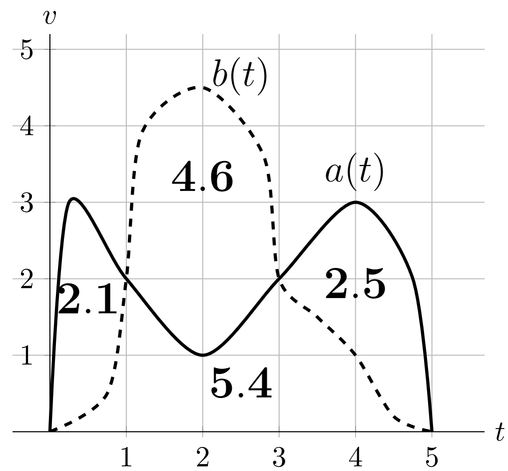
Therfore,

$$\begin{aligned} \int_4^6 \frac{f'(x)}{(f(x))^2 + f(x)}\, dx &= (\ln|f(6)| - \ln|f(6)+1|) - (\ln|f(4)| - \ln|f(4)+1|) \\ &= \ln(5) - \ln(6) + \ln(2) \\ &\approx 0.510825 \end{aligned}$$

Answer: d. $\ln(5) - \ln(6) + \ln(2) \approx 0.510825$

2. [9 points]

When Alejandra and Brontel were children they spent summer mornings chasing birds in flight. One memorable day they encountered an owl. The following graph shows the velocities $a(t)$ of Alejandra (solid) and $b(t)$ of Brontel (dashed), measured in meters per second, t seconds after the owl took off. The area of each region is given.



a. [1 point] How far (in meters) do Alejandra and Brontel chase the owl?

Solution: Summing the areas under either curve gives a total distance of 10 m.

b. [5 points] Suppose the owl ascends to a height of h meters according to $h(t) = \sqrt{t}$ where t is seconds since it went airborne. Let $L(h)$ be the number of meters Alejandra is ahead of Brontel when the owl is h meters above ground. Write an expression for $L(h)$ involving integrals and compute $L'(2)$.

Solution: The owl is h meters above the ground at time $t = h^2$. Thus,

$$L(h) = \int_0^{h^2} a(t) - b(t)\, dt.$$

We compute $L'(h)$ using the second fundamental theorem of calculus.

$$L'(h) = 2ha(h^2) - 2hb(h^2).$$

So we have

$$L'(2) = 2 \cdot 2a(4) - 2 \cdot 2b(4) = 4 \cdot 3 - 4 \cdot 1 = 8,$$

where we get the values of $a(2)$ and $b(2)$ from the graph.

c. [3 points] The next bird to pass is a dove. This time Alejandra runs twice as fast and Brontel runs three times as fast as they did when chasing the owl. How much faster (in m/s) is Brontel than Alejandra on average in the first 5 seconds?

Solution: The integral that represents this average is

$$\frac{1}{5}\int_0^5 3b(t) - 2a(t)\, dt = \frac{3}{5}\int_0^5 b(t)\, dt - \frac{2}{5}\int_0^5 a(t)\, dt.$$

Each of these integrals is equal to 10 as we see from the graph. Hence

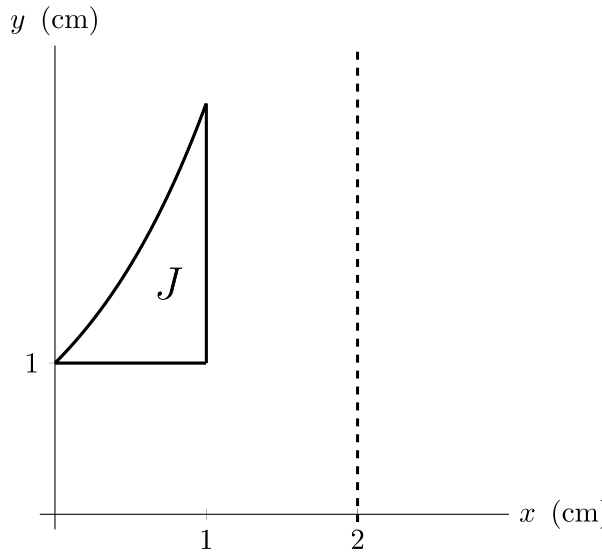
$$\frac{1}{5}\int_0^5 3b(t) - 2a(t)\, dt = \frac{10}{5} = 2.$$

Answer: **2 m/s**

3. [10 points]

Debra McQueath hooked you up with an interview at `Print.juice`. Being a legitimate tech start-up, the `Print.juice` interview consists of answering technical questions on the spot. Debra gave you the following questions for practice.

The region J is a common `Print.juice` shape. It is bounded by $x = 1$, $y = 1$, and $y = e^x$.



a. [3 points] First, consider the solid with base J and square cross sections perpendicular to the x-axis. If the density of the solid is a function of the x-coordinate $a(x)$ g/cm^3, write an integral that represents the total mass of the solid in grams.

Solution: The height of a cross-section is $e^x - 1$, thus the total mass is

$$\int_0^1 a(x)(e^x - 1)^2\,dx.$$

For b. and c., consider the solid made by rotating J around the line $x = 2$.

b. [3 points] If the density of the solid is a function of the y-coordinate $b(y)$ g/cm^3, write an integral that represents the total mass of the solid in grams.

Solution: Using the washer method we compute the total mass to be

$$\int_1^e b(y)\pi((2 - \ln(y))^2 - 1^2)\,dy.$$

c. [4 points] If the density of the solid is a function of the distance r cm from the axis of rotation $c(r)$ g/cm^3, write an integral that represents the total mass of the solid in grams.

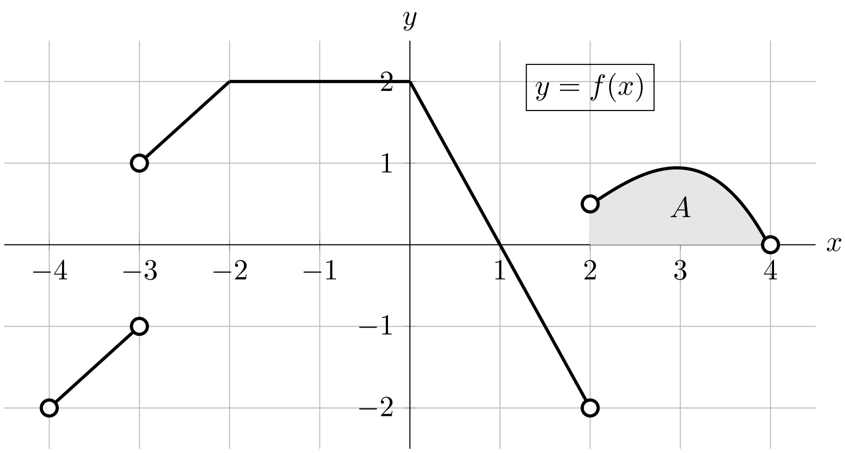
Solution: Using the shell method we can either compute the mass in terms of x or r. In terms of r we get

$$\int_1^2 c(r)2\pi r(e^{2-r} - 1)\,dr,$$

and in terms of x we get

$$\int_0^1 c(2 - x)2\pi(2 - x)(e^x - 1)\,dx.$$

4. [10 points] The entire graph of the function $f(x)$ is given below. Note that $f(x)$ is piecewise linear on $(-4, 2)$, and the area of the shaded region A is 1.5.



a. [2 points] Let $F(x)$ be the continuous antiderivative of $f(x)$ passing through $(2, 1)$. Circle all of the x-coordinates listed below at which $F(x)$ appears to have an inflection point.

$x = -3$

$x = 1$

$x = 2$

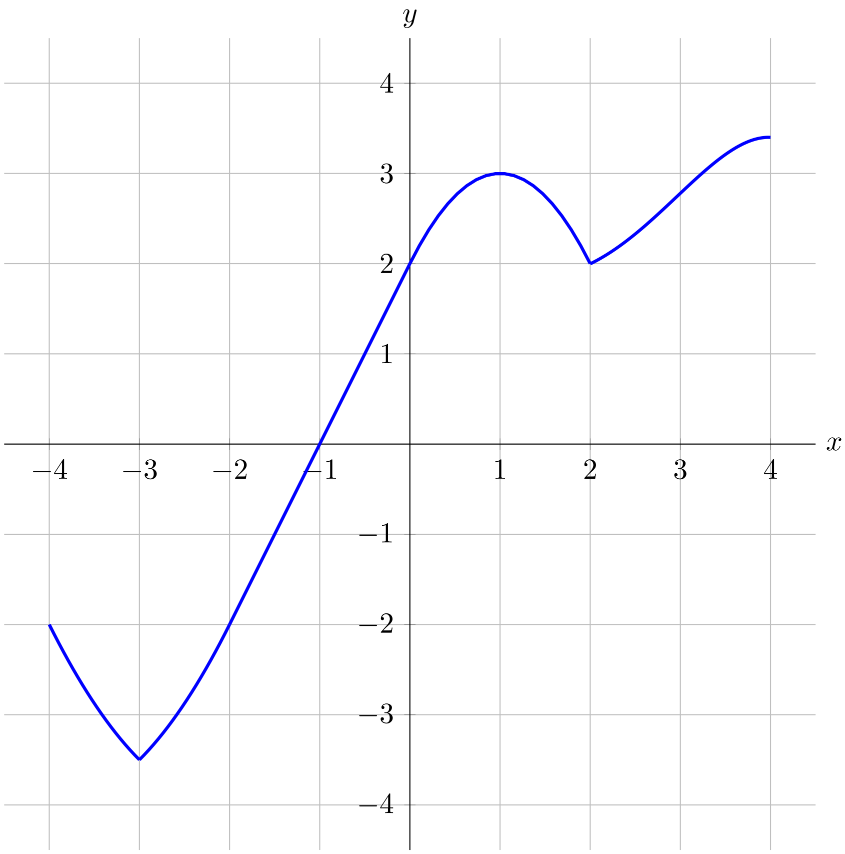
$x = 3$

NONE OF THESE

b. [8 points] On the axes to the right, sketch a graph of the function $G(x)$, a continuous antiderivative of $f(x)$ given on $(-3, 2)$ by

$$G(x) = \int_{-1}^{x} f(t)\, dt.$$

Make sure that local extrema and concavity are clear. If there are features that are difficult for you to draw, indicate these on your graph.



5. [9 points] Tammy Toppel is directing a performance art piece at the community center. She fills a large cone with sand and cuts a small hole in the bottom. Gerd Hömf was hired from a temp agency to stand behind the scenes and steadily lift the cone with an elaborate pulley system, letting the sand slowly spill onto the stage.

a. [2 points] The filled cone starts with a total mass of 40 kilograms and spills sand at a constant rate of 1/2 a kilogram per second once it is lifted. Tammy wants Gerd to lift the cone at a constant rate of r meters per second. Find a formula for the mass $M(h)$, in kilograms, of the cone when it is h meters above the stage.

Solution: The relation between the height h of the cone and time t is $h = rt$, where r is the rate at which Gerd lifts. So $t = h/r$. The mass as a function of time is given by $40 - \frac{1}{2}t$, hence

$$M(h) = 40 - \frac{h}{2r}.$$

b. [4 points] Gerd lifts the cone until it reaches a height of 20 meters above the stage. Write an integral which represents the work (measured in Joules) done by Gerd while lifting the cone. The integral may include the rate r at which Gerd lifts and g the acceleration (in $\mathrm{m/s^2}$) due to gravity.

Solution: The work done by Gerd lifting the cone to a height of h is given by

$$\int_0^{20} gM(h)\,dh = \int_0^{20} g\left(40 - \frac{h}{2r}\right)\,dh$$

c. [3 points] There's one catch: Gerd's contract strictly prohibits him from exerting more than $500g$ Joules of work, where g is the acceleration due to gravity. At what rate r (in m/s) should Tammy ask Gerd to lift in order to not violate his contract and to get the cone lifted as quickly as possible?

Solution: We set the integral from the previous part equal to $500g$ and solve for r.

$$\begin{aligned} 500g &= \int_0^{20} g\left(40 - \frac{h}{2r}\right)\,dh \\ &= \left(40gh - \frac{h^2}{4r}\right)\Bigg|_0^{20} \\ &= 800g - \frac{400}{4r} - 0 \\ &= g\left(800 - \frac{100}{r}\right). \end{aligned}$$

Dividing both sides by g and multiplying by r we have

$$500r = 800r - 100 \Longrightarrow r = \frac{1}{3}.$$

Answer: $r =$ **1/3**

6. [7 points] For each of the questions below, circle **all** of the available correct answers. You must circle at least one choice to receive any credit. No credit will be awarded for unclear markings. No justification is necessary.

a. [3 points] Which $F(x)$ are antiderivatives of $f(x) = e^{x^2}$ with $F(3) = 5$ for $x > 0$?
Note: due to a typo in the original exam (corrected here), a student's answer to option IV did not impact their score.

I. $F(x) = \int_0^{x^2} e^u\, du + 5$

II. $F(x) = \int_3^{x} 5e^{u^2}\, du$

III. $F(x) = \frac{1}{x^2}e^{x^2} + 5$

IV. $F(x) = \int_{x^2}^{9} -\frac{1}{2\sqrt{u}}e^u\, du + 5$

V. $F(x) = \int_3^{x} e^{u^2}\, du + 5$

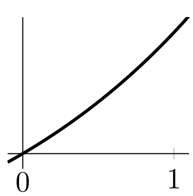
VI. $F(x) = \frac{e^{x^2}}{2x} - \frac{e^9}{6} + 5$

b. [2 points] Suppose $f(x)$ is an odd function. Which values of b make the following equation true?

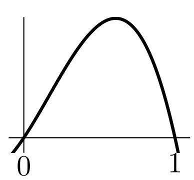
$$\int_{-\pi}^{b} \sin(f(x))\, dx = 0$$

I. $b = -\pi$ II. $b = 0$ III. $b = \pi$ IV. $b = \frac{3\pi}{2}$ V. $b = 2\pi$

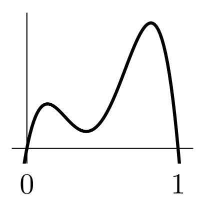
c. [2 points] Which of the following could be the graph of $f(x) = \int_x^{x^3} e^{\sqrt[3]{u}}\, du$?



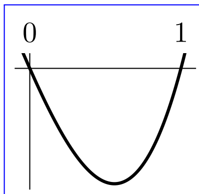
I.



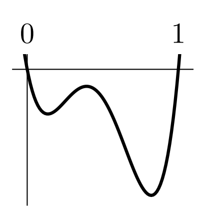
II.



III.



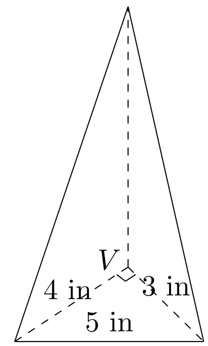
IV.



V.

7. [10 points]

Ms. Parth made a pyramid for her niece and nephew. The pyramid is 10 inches tall and the base has the shape of a right triangle. When the pyramid is sitting on the table it looks like the figure to the right. The three angles at vertex V are right angles. (Dashed lines are not visible from this point of view. The figure may not be drawn to scale.)



a. [6 points] Write an integral that represents the total volume of the pyramid in cubic inches and evaluate it.

Solution: Let h represent inches from the top of the pyramid. A cross-section h inches from the top of the pyramid and parallel to the table is a right triangle similar to the base. Using similar triangles we find that the cross-section has side lengths $\frac{3}{10}h$ and $\frac{4}{10}h$. Hence the integral representing its volume is

$$\text{Volume} = \int_0^{10} \frac{1}{2}\left(\frac{3}{10}\right)\left(\frac{4}{10}\right) h^2\, dh$$
$$= \frac{6}{10^2}\int_0^{10} h^2\, dh.$$

Then the fundamental theorem of calculus gives

$$\text{Volume} = \frac{6}{10^2}\cdot\frac{h^3}{3}\bigg|_0^{10} = 20\,\text{in}^3.$$

If y represents inches above the table, then the integral will be

$$\text{Volume} = \int_0^{10} \frac{1}{2}\left(\frac{3}{10}\right)\left(\frac{4}{10}\right)(10-y)^2\, dy.$$

b. [4 points] The children fail to share the pyramid, so Ms. Parth decides to cut it parallel to the table into two pieces of equal volume. How many inches H from the **top** of the pyramid should Ms. Parth cut? Round your answer to the nearest tenth of an inch.

Solution: Since the volume of the pyramid is 20 in^3, we want to find H satisfying

$$10 = \int_0^{H} \frac{1}{2}\left(\frac{3}{10}\right)\left(\frac{4}{10}\right) h^2\, dh.$$

Using the fundamental theorem of calculus this becomes

$$10 = \frac{2}{100}H^3 \longrightarrow H = \frac{10}{\sqrt[3]{2}} \approx 7.937.$$

8. [14 points] Let $g(x)$ be a differentiable function with domain $(-1, 10)$ where some values of $g(x)$ and $g'(x)$ are given in the table below. Assume that all local extrema and critical points of $g(x)$ occur at points given in the table.

x	0	1	2	3	4	5	6	7	8
$g(x)$	2.0	3.3	5.7	6.8	6.0	4.3	2.4	0.2	-4.9
$g'(x)$	2.8	2.5	2.0	0.0	-1.4	-1.9	-1.6	-3.0	-8.1

a. [3 points] Estimate $\int_0^8 g(x)\,dx$ using RIGHT(4). Write out each term in your sum.

Solution: With 4 rectangles the width of each is $\Delta x = \frac{8-0}{4} = 2$. Then

$$\begin{aligned} \text{RIGHT}(4) &= g(2)\Delta x + g(4)\Delta x + g(6)\Delta x + g(8)\Delta x \\ &= (5.7 + 6.0 + 2.4 - 4.9) \cdot 2 \\ &= 18.4 \end{aligned}$$

Answer: **18.4**

b. [4 points] Approximate the area of the region between $g(x)$ and the function $f(x) = x + 2$ for $0 \le x \le 4$, using MID(n) to estimate any integrals you use. Use the greatest number of subintervals possible, and write out each term in your sums.

Solution: The function $g(x)$ is concave down on $[0, 4]$, so $g(x)$ is greater than or equal to the linear function $f(x)$ on this interval. The integral to compute this area is

$$\int_0^4 g(x) - f(x)\,dx = \int_0^4 g(x)\,dx - \int_0^4 x + 2\,dx.$$

Since $f(x)$ is linear, we get the same answer whether we use MID to approximate $\int_0^4 g(x) - f(x)\,dx$ or just $\int_0^4 g(x)\,dx$ and compute $\int_0^4 f(x)\,dx$ exactly. In either case, we can use at most 2 subintervals and $\Delta x = 2$.
If we compute MID(2) for $\int_0^4 g(x) - f(x)\,dx$, we get

$$\begin{aligned} \text{MID}(2) &= (g(1) - f(1))\Delta x + (g(3) - f(3))\Delta x \\ &= ((3.3 - 3) + (6.8 - 5))2 \\ &= (.3 + 1.8)2 \\ &= 4.2. \end{aligned}$$

If we compute $\int_0^4 f(x)\,dx = 16$ and then compute MID(2) for $\int_0^4 g(x)\,dx$ we get

$$\begin{aligned} \text{MID}(2) &= g(1)\Delta x + g(3)\Delta x \\ &= (3.3 + 6.8)2 \\ &= 20.2. \end{aligned}$$

Then we get $20.2 - 16 = 4.2$ for the total area.

Answer: **4.2**

c. [3 points] Is your answer to **b.** an overestimate, an underestimate, or is there not enough information to tell? Briefly justify your answer.

Solution: Since we are only given a table and not told that the concavity does not change between points, we technically **do not have enough information** to answer this question.
Had it been the case that $g'(x)$ has no critical points aside from those in the table, it would follow that $g(x)$ is concave down, because $g'(x)$ would be decreasing on the given interval. Since $f(x)$ is linear, the concavity of $g(x) - f(x)$ would also be concave down. In that case, MID(2) would be an **overestimate.**
Credit was awarded for both of these answers.

d. [4 points] Write an integral giving the arc length of $y = g(x)$ between $x = 2$ and $x = 8$. Estimate this integral using TRAP(2). Write out each term in your sum.

Answer: Integral: $\int_2^8 \sqrt{1+g'(x)^2}\,dx$

Solution: The arc length is given by the integral

$$\text{Arc length} = \int_2^8 \sqrt{1+g'(x)^2}\,dx.$$

The width of our trapezoids is $\Delta x = \frac{8-2}{2} = 3$.
If we compute the areas of the trapezoids directly we get

$$\begin{aligned}\text{TRAP}(2) &= \left(\frac{\sqrt{1+g'(2)^2}+\sqrt{1+g'(5)^2}}{2}\right)\Delta x + \left(\frac{\sqrt{1+g'(5)^2}+\sqrt{1+g'(8)^2}}{2}\right)\Delta x\\ &\approx (2.1915795 + 5.1542930)3\\ &\approx 22.0376175.\end{aligned}$$

If we compute LEFT(2) and RIGHT(2) first and then take an average we get

$$\begin{aligned}\text{LEFT}(2) &= \sqrt{1+g'(2)^2}\Delta x + \sqrt{1+g'(5)^2}\Delta x\\ &\approx (4.3831590)3\\ &\approx 13.1494770.\end{aligned}$$

$$\begin{aligned}\text{RIGHT}(2) &= \sqrt{1+g'(5)^2}\Delta x + \sqrt{1+g'(8)^2}\Delta x\\ &\approx (10.3085860)3\\ &\approx 30.9257580.\end{aligned}$$

Then

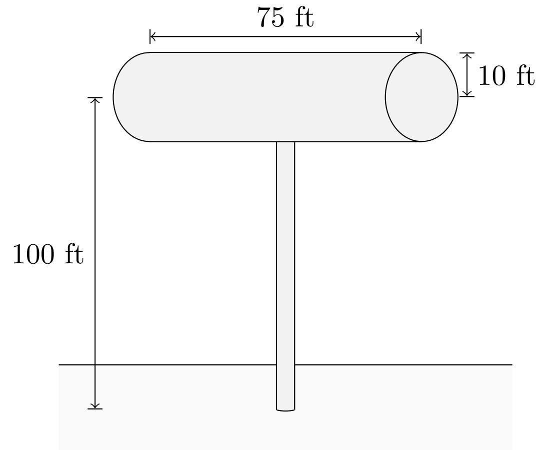
$$\text{TRAP}(2) = \frac{1}{2}(\text{LEFT}(2) + \text{RIGHT}(2)) \approx 22.0376175.$$

Answer: TRAP(2)= **22.0376175**

9. [9 points]

De'von Baptiste is a shrewd industrialist. When energy costs are low, De'von pumps purified muck (which he gets *for free* from the city) into very tall tanks. In this way he stores cheap potential energy. Someday, when energy prices soar, Mr. Batiste will convert it all back into useful kinetic energy at a great profit.

His tanks are cylinders 75 ft long with radius 10 ft. The center of a tank is 100 ft above the ground. Purified muck has a density of 800 pounds/ft^3.



a. [3 points] What is the area, in square feet, of a cross-section parallel to the ground taken y feet above the **center** of the tank?

Solution: The cross-sections are rectangles with a length of 75 ft and a width $w(y)$ which depends on y. Using the Pythagorean Theorem we find that

$$10^2 = y^2 + (w(y)/2)^2 \longrightarrow w(y) = 2\sqrt{100 - y^2} = \sqrt{400 - 4y^2}.$$

Hence the area of a cross-section is

$$\text{Area of cross-section} = 75w(y) = 75 \cdot 2\sqrt{100 - y^2} = 150\sqrt{100 - y^2}.$$

Answer: $150\sqrt{100 - y^2}$

b. [6 points] Write an integral which represents the total work (in foot-pounds) required to fill one of De'von Batiste's tanks with purified muck. **Do not evaluate this integral.**

Solution: If we consider the tank after its filled, we can compute the work required to get each slice of muck y feet above the center of the tank from the ground to its height at $100 + y$ ft above the ground. If $A(y) = 300\sqrt{100 - y^2}$ is the area of a cross-section y feet above the center of the tank, then the total work is

$$\begin{aligned}\text{Total work} &= \int_{-10}^{10} (\text{density})(\text{distance})(\text{slice volume}) \\ &= \int_{-10}^{10} 800(100 + y)A(y)\,dy \\ &= \int_{-10}^{10} 800(100 + y)150\sqrt{100 - y^2}\,dy\end{aligned}$$

10. [8 points] For each of the questions below, circle all of the available correct answers. Circle NONE OF THESE if none of the available choices are correct. You must circle at least one choice to receive any credit. No credit will be awarded for unclear markings. No justification is necessary.

a. [3 points] Which of the following integrals are equal to $\lim_{n\to\infty} \sum_{k=0}^{n-1} \left(3 + \frac{2k}{n}\right)^4 \cdot \frac{2}{n}$?

i. $\int (1+kx)^4 x\, dx$

ii. $\int_3^5 x^4\, dx$

iii. $\int_0^{n-1} \left(3 + \frac{2x}{n}\right)^4 \cdot \frac{2}{n}\, dx$

iv. $\int_2^3 x^4\, dx$

v. $\int_0^2 (3+x)^4\, dx$

vi. NONE OF THESE

b. [3 points] Which of the following expressions give the volume of the solid made by rotating around the y-axis the region bounded by $y = \sin(x)$, $y = 0$, and $x = \frac{\pi}{2}$?

i. $\int_0^{\pi/2} \pi \left(\frac{\pi}{2} - \sin(x)\right)^2\, dx$

ii. $\int_0^{\pi/2} \pi \left(\left(\frac{\pi}{2}\right)^2 - \sin^2(x)\right)\, dx$

iii. $\int_0^{\pi/2} 2\pi x \sin(x)\, dx$

iv. $\int_0^{\pi/2} \pi \sin^2(x)\, dx$

v. $\int_0^1 \pi \left(\frac{\pi}{2} - \arcsin(y)\right)^2\, dy$

vi. $\int_0^1 \pi \left(\left(\frac{\pi}{2}\right)^2 - (\arcsin(y))^2\right)\, dy$

vii. $\int_0^1 2\pi y \arcsin(y)\, dy$

viii. NONE OF THESE

c. [2 points] Let $f(x)$ be a function that is increasing on $(-3, 3)$, concave up on $(0, 3)$, and has a point of inflection at $x = 0$. Consider the approximations for $\int_{-2}^{2} f(x)\, dx$ given by LEFT(n) and TRAP(n). Which of the following statements **must** be true?

i. $\text{TRAP}(n) < \int_{-2}^{2} f(x)\, dx$

ii. $\text{TRAP}(n) > \int_{-2}^{2} f(x)\, dx$

iii. TRAP(n) is neither an overestimate nor an underestimate for $\int_{-2}^{2} f(x)\, dx$.

iv. $\text{LEFT}(n) < \int_{-2}^{2} f(x)\, dx$

v. $\text{LEFT}(n) > \int_{-2}^{2} f(x)\, dx$

vi. $\text{LEFT}(n) = 0$

vii. NONE OF THESE