Math 116 — First Midterm — February 10, 2020

EXAM SOLUTIONS

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 14 pages including this cover. Do not separate the pages. If pages do become separated, write your UMID on every page and point this out to your instructor when you hand in the exam.
4. There are 11 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
5. Note that the back of every page of the exam is blank, and, if needed, you may use this space for scratchwork. Clearly identify any of this work that you would like to have graded.
6. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
7. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it.
8. The use of any networked device while working on this exam is not permitted.
9. You may use any one calculator that does not have an internet or data connection except a TI-92 (or other calculator with a "qwerty" keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a single $3'' \times 5''$ notecard.
10. For any graph or table that you use to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how you used the graph or table to find the answer.
11. Include units in your answer where that is appropriate.
12. Problems may ask for answers in *exact form*. Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is not.
13. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones, earbuds, and smartwatches. Put all of these items away.
14. You must use the methods learned in this course to solve all problems.
15. The pdf of a normal distribution with mean μ and standard deviation $\sigma > 0$ is $\dfrac{e^{-(x-\mu)^2/(2\sigma^2)}}{\sigma\sqrt{2\pi}}$.

Problem	Points	Score
1	12	
2	10	
3	7	
4	5	
5	10	
6	9	

Problem	Points	Score
7	6	
8	12	
9	10	
10	7	
11	12	
Total	100	

1. [12 points] The two continuous functions $g(x)$ and $h(x)$ have the following properties.

- $\int_{-2}^{4} g(t)\, dt = 11$
- $\int_{-1}^{2} g(t)\, dt = 5$
- $\int_{-0.5}^{1} g(t)\, dt = 3$
- $g(x) = 7$ on the interval $[4, 6]$
- $H(5) - H(1) = -3$, where $H(x)$ is an antiderivative of $h(x)$
- $\int_{-2}^{2} h(x)\, dx = 6.$
- $\int_{5}^{14} h(x)\, dx = 15$
- $\int_{-\infty}^{2} h(x)\, dx = 20.$

Calculate the following values. Write "DIVERGES" if appropriate.
If there is not enough information provided to find the exact value, write "NOT ENOUGH INFO."
You are not required to show your work on this problem. However, limited partial credit may be awarded based on work shown.

a. [3 points] $\int_{-2}^{6} g(x)\, dx$

Solution:
$$\int_{-2}^{6} g(x)\, dx = \int_{-2}^{4} g(x)\, dx + \int_{4}^{6} g(x)\, dx = 11 + \int_{4}^{6} 7\, dx = 11 + 2(7) = 25$$

Answer: 25

b. [3 points] $\int_{-1}^{2} g(2x)\, dx$

Solution: Using substitution with $w = 2x$ (so $dw = 2\, dx$), we find
$$\int_{-1}^{2} g(2x)\, dx = \frac{1}{2}\int_{-2}^{4} g(w)\, dw = \frac{11}{2} = 5.5$$

Answer: 5.5

c. [3 points] The average value of $h(x)$ on the interval $[1, 5]$

Solution: With H as in the bullet points above, the formula for average value of a function gives
$$\frac{1}{4}\int_{1}^{5} h(x)\, dx = \frac{1}{4}\left(H(5) - H(1)\right) = \frac{-3}{4} = -0.75$$

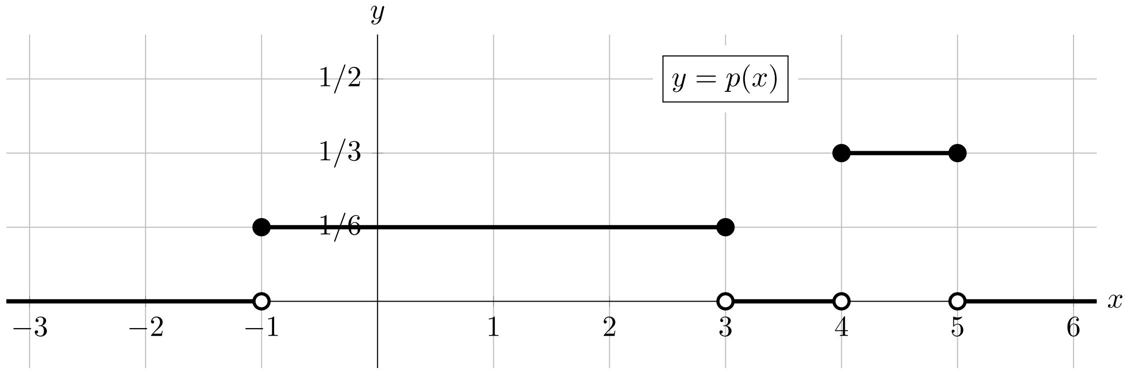
Answer: -0.75

d. [3 points] $\int_{-2}^{\infty} h(-x)\, dx$

Solution:
$$\begin{aligned}\int_{-2}^{\infty} h(-x)\, dx &= \lim_{b\to\infty}\int_{-2}^{b} h(-x)\, dx \\ &= \lim_{b\to\infty}\int_{2}^{-b} -h(w)\, dw \qquad \text{(substitution with } w = -x) \\ &= \lim_{b\to\infty}\int_{-b}^{2} h(w)\, dw \\ &= \int_{-\infty}^{2} h(w)\, dw = 20\end{aligned}$$

Answer: 20

2. [10 points] The graph of a probability density function (pdf) $p(x)$ is shown below.



Note: Your answers to **a.** and **b.** should not include integral signs, variables, or function names.

a. [3 points] Find the median value of a quantity with pdf $p(x)$.

Solution: Let T be the median value. Since $\int_{-\infty}^{-1} p(x)\,dx = 0$ and $\int_{-1}^{3} p(x)\,dx = 2/3$, we see that $-1 < T < 3$. Using rectangles (or $\int_{-1}^{T} p(x)\,dx$) we find $0.5 = (1/6)(T - (-1)) = \frac{T+1}{6}$ so $T + 1 = 3$ and $T = 2$.

Answer: The median value is 2

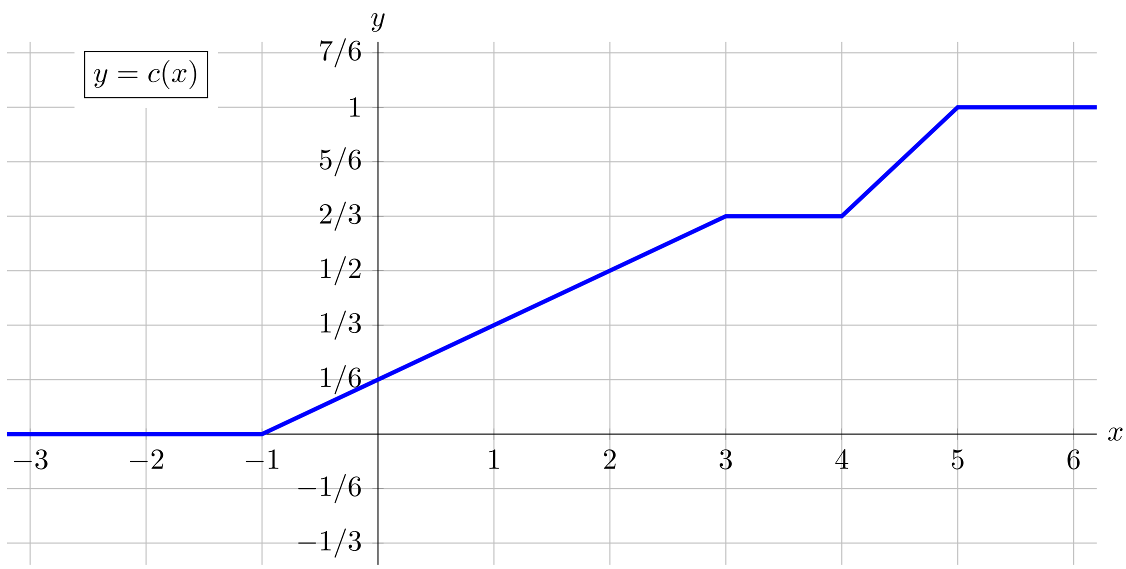
b. [3 points] Find the mean value of a quantity with pdf $p(x)$.

Solution:

$$\text{mean value} = \int_{-\infty}^{\infty} xp(x)\,dx = \int_{-1}^{3} \frac{x}{6}\,dx + \int_{4}^{5} \frac{x}{3}\,dx = \frac{x^2}{12}\Big|_{-1}^{3} + \frac{x^2}{6}\Big|_{4}^{5}$$
$$= \frac{9-1}{12} + \frac{25-16}{6} = \frac{8}{12} + \frac{9}{6} = \frac{13}{6}$$

Answer: The mean value is $\frac{13}{6}$

c. [4 points] If $c(x)$ is the cumulative distribution function corresponding to the pdf function $p(x)$ above, sketch a graph of $y = c(x)$ on the axes below. Pay careful attention to where your graph is differentiable, increasing/decreasing, and concave up/concave down.



3. [7 points] The function g defined by $g(x) = \ln(x^2+1)$ is differentiable for all x in $(-\infty, \infty)$. For all $x > 0$, the function $B(x) = \frac{1}{x}\int_0^x \ln(t^2+1)\,dt$ gives the average value of $g(x)$ over the interval $[0, x]$.

Note: Your answers may require one or more integral signs. However, neither the letter g nor the letter B should appear in your answers.

a. [4 points] Calculate $B'(x)$.

Solution: Using the product rule and the Construction Theorem (aka Second Fundamental Theorem of Calculus), we have

$$B'(x) = \left(\frac{d}{dx}\left(\frac{1}{x}\right)\right)\cdot\int_0^x \ln(t^2+1)\,dt + \frac{1}{x}\cdot\frac{d}{dx}\left(\int_0^x \ln(t^2+1)\,dt\right)$$
$$= \frac{-\int_0^x \ln(t^2+1)\,dt}{x^2} + \frac{\ln(x^2+1)}{x} = \frac{x\ln(x^2+1) - \int_0^x \ln(t^2+1)\,dt}{x^2}$$

Note that although it is possible to compute the integral $\int_0^x \ln(t^2+1)\,dt$, it is not necessary to do so here.

Answer: $B'(x) =$ $\dfrac{x\ln(x^2+1) - \int_0^x \ln(t^2+1)\,dt}{x^2}$

b. [3 points] Write a formula for the average value of g' over the interval $[0, x]$.

Solution: (for $x \neq 0$)

$$\text{Average value of } g' \text{ over } [0,x] = \frac{1}{x-0}\int_0^x g'(t)\,dt = \frac{1}{x}\left(g(x) - g(0)\right)$$
$$= \frac{\ln(x^2+1) - \ln(0+1)}{x} = \frac{\ln(x^2+1)}{x}$$

Answer: Average value of g' over $[0, x]$ equals $\dfrac{\ln(x^2+1)}{x}$

4. [5 points] Determine whether the integral $\displaystyle\int_0^3 \frac{1}{x^{\pi/4}}\,dx$ converges or diverges.

- If the integral converges, circle "Converges", find its exact value, and write the exact value on the answer blank provided.
- If the integral diverges, circle "Diverges" and carefully justify your answer.

In either case, you must show all your work and use proper notation. Evaluation of integrals must be done **without using a calculator**.

Note that $\displaystyle\frac{1}{x^{\pi/4}} = x^{-\pi/4}$.

Circle one:

$\displaystyle\int_0^3 \frac{1}{x^{\pi/4}}\,dx$ **converges to** $\displaystyle\frac{3^{-(\pi/4)+1}}{-(\pi/4)+1}$ or $\displaystyle\int_0^3 \frac{1}{x^{\pi/4}}\,dx$ **diverges**

Solution:

$$\int_0^3 \frac{1}{x^{\pi/4}}\,dx = \lim_{a\to 0^+}\int_a^3 \frac{1}{x^{\pi/4}}\,dx = \lim_{a\to 0^+}\int_a^3 x^{-\pi/4}\,dx$$

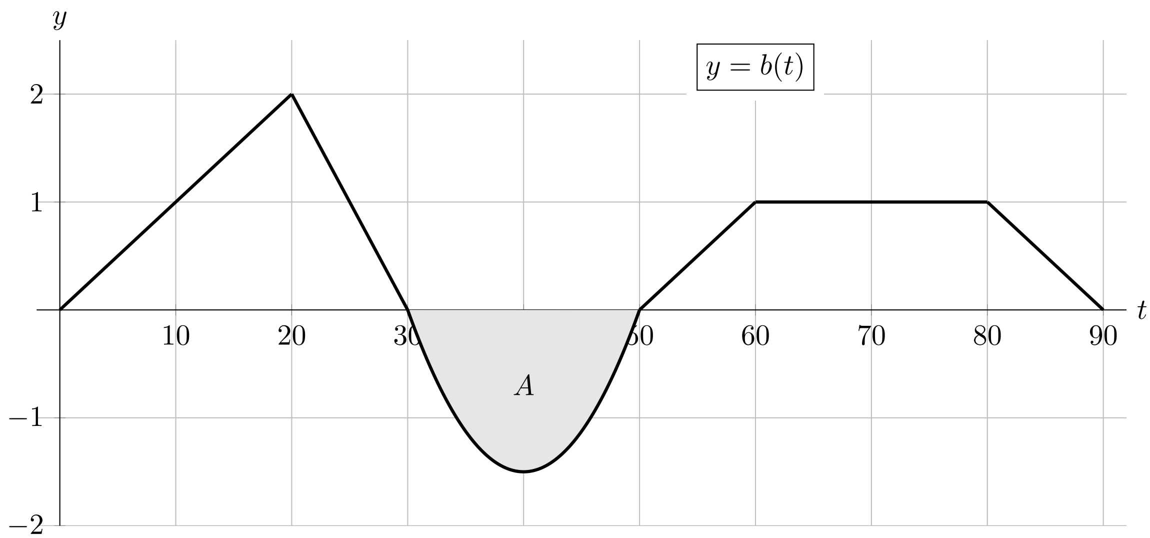
$$= \lim_{a\to 0^+} \left.\frac{x^{-(\pi/4)+1}}{-(\pi/4)+1}\right|_{x=a}^{x=3}$$

$$= \lim_{a\to 0^+}\left[\frac{3^{-(\pi/4)+1}}{-(\pi/4)+1} - \frac{a^{-(\pi/4)+1}}{-(\pi/4)+1}\right]$$

$$= \frac{3^{-(\pi/4)+1}}{-(\pi/4)+1} \qquad \text{(since } -(\pi/4)+1 \text{ is positive)}$$

5. [10 points] After a long day filming a space walk scene for a movie, an actress attends a loud rock concert. Her fancy new watch monitors her stress levels during the concert. Among other data, it outputs the rate of change in her heart rate. Let $b(t)$ be the rate of change in her heart rate (in beats/minute^2) t minutes after the concert begins.

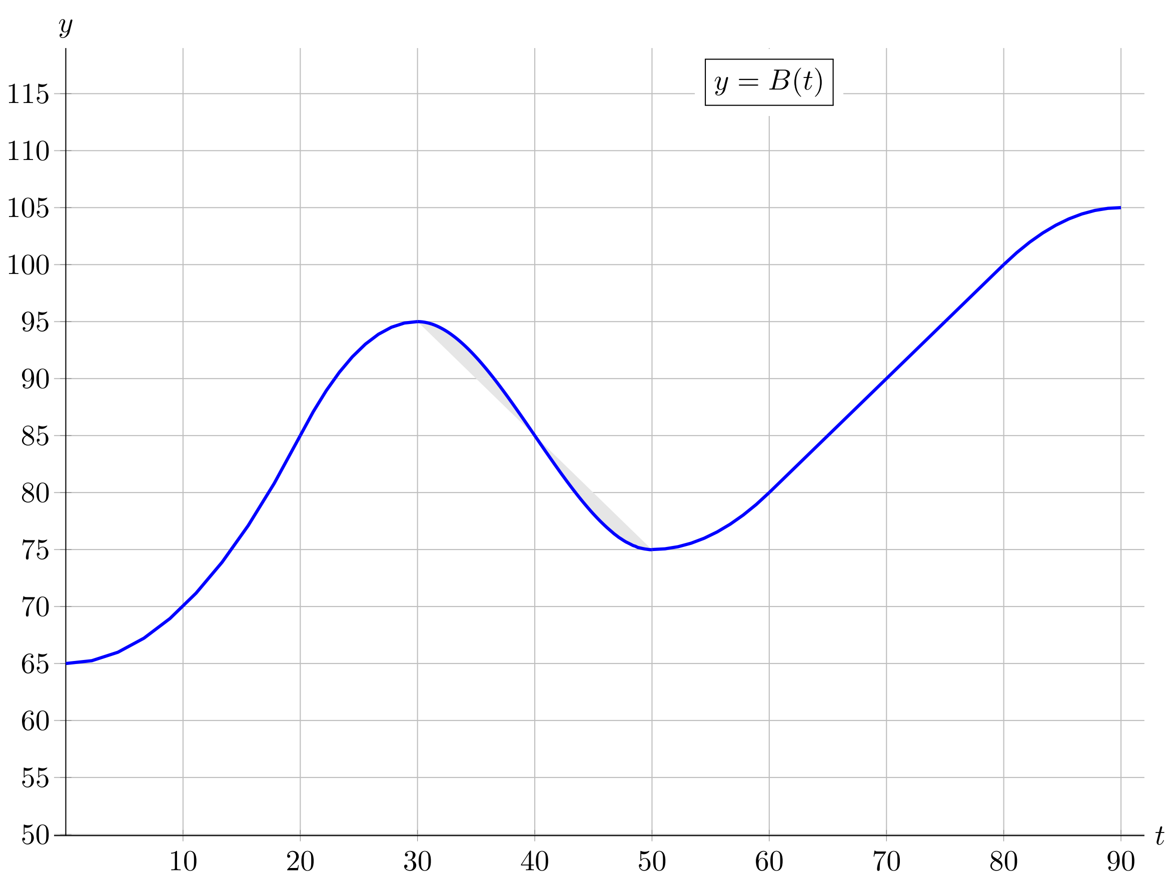
A graph of $b(t)$ is given below. The area of the shaded region A is 20.



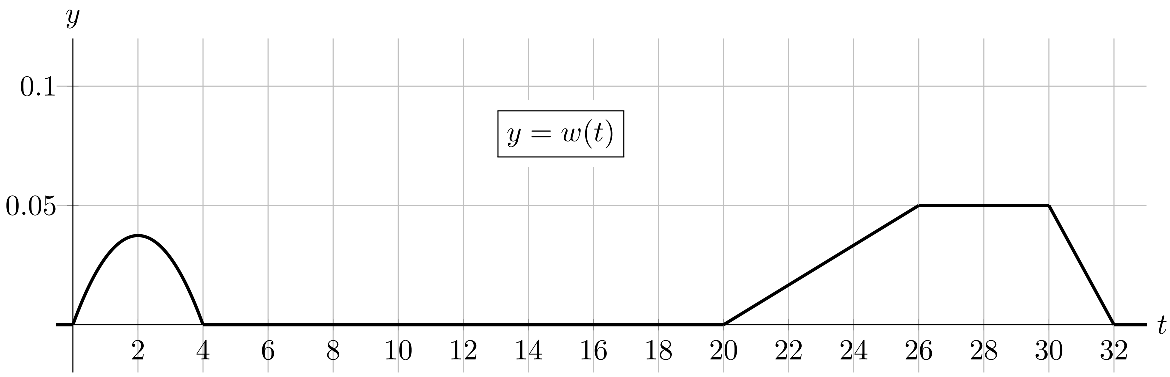
Let $B(t)$ be the actress's heart beat (in beats per minute) t minutes after the concert begins. Suppose that her heart rate 60 minutes into the concert is 80 beats/minute.

Sketch a detailed graph of $B(t)$ for $0 \le t \le 90$. Pay careful attention to where your graph is differentiable, increasing/decreasing, and concave up/concave down.

Solution:



6. [9 points] As part of his exercise routine, a man goes for walks of various lengths of time. The lengths of the man's walks, where t is measured in minutes, are described by the density function $w(t)$. A portion of the graph of $w(t)$ is shown below.



a. [3 points] Complete the following English sentence:

The fraction of the man's walks that are between 20 and 28 minutes long is . . .

Solution: 0.25 (or 25%)

b. [3 points] Circle the ONE sentence below that BEST corresponds to the mathematical statement $w(3) \approx 0.028$.

i. Approximately 3% of the man's walks last between 0.028 and 1.028 minutes.

ii. Approximately 1.4% of the man's walks last between 3 and 3.5 minutes.

iii. Approximately 28% of the man's walks last between 3 and 4 minutes.

iv. Approximately 2.8% of the man's walks last exactly 3 minutes.

v. Approximately 3% of the man's walks last approximately 2.8 minutes.

c. [3 points] Does the man take any walks that last longer than 32 minutes? Explain.

Circle one: YES NO NOT ENOUGH INFORMATION

Explanation:

Solution: We see that

$$\int_0^{32} w(t)\,dt = \int_0^4 w(t)\,dt + \int_{20}^{32} w(t)\,dt = \int_0^4 w(t)\,dt + 0.4$$

$$\leq \text{MID}(1) + 0.4 \quad \text{(where MID(1) is an (over)estimate of } \int_0^4 w(t)\,dt \text{ as this portion of the graph of } w(t) \text{ is concave down)}$$

$$< 0.05(4) + 0.4 = 0.6$$

So less that 60% of the man's walks are represented by the portion of the graph shown above. Since walks cannot last for a negative length of time, at least 40% of the man's walks last longer than 32 minutes.

7. [6 points] A new edition of an old video game features a rocket which blasts off into space along a staight-line path. As in the earlier edition of the video game, at $t = 10$ seconds after the rocket engines ignite, the rocket detaches from the platform and lifts off. The game designers slightly altered the speed function in the new edition to

$$r(t) = \frac{1}{t^{4/5}(1+t^{2/5})}\text{km/second},$$

where t is measured in seconds after the engines ignite so the formula for $r(t)$ given above is valid for $t \geq 10$.

a. Assuming time in the video game goes on forever, write an expression involving an integral that represents the distance from the launchpad that the rocket approaches as time goes on.

Solution:

$$\int_{10}^{\infty} \frac{1}{t^{4/5}(1+t^{2/5})}\, dt$$

Answer: $\int_{10}^{\infty} \frac{1}{t^{4/5}(1+t^{2/5})}\, dt$

b. Determine whether the answer to part **a.** converges or diverges.

- If the integral converges, circle "Converges", find its exact value, and write the exact value on the answer blank provided.
- If the integral diverges, circle "Diverges" and carefully justify your answer.

In either case, you must show all your work and use proper notation. Evaluation of integrals must be done **without using a calculator**.

Hint: let $w = t^{1/5}$.

Circle one: **Converges to** $\frac{5\pi}{2} - 5\arctan(10^{1/5})$ or **Diverges**

Solution: We have

$$\int_{10}^{\infty} \frac{1}{t^{4/5}(1+t^{2/5})}\, dt =$$

$$= \lim_{b\to\infty} \int_{10}^{b} \frac{1}{t^{4/5}(1+t^{2/5})}\, dt.$$

We now substitute with $w = t^{1/5}$ and $dw = \frac{1}{5}t^{-4/5}\, dt$ (or equivalently $5\, dw = \frac{1}{t^{4/5}}\, dt$).

$$= \lim_{b\to\infty} \int_{10^{1/5}}^{b^{1/5}} \frac{5}{1+w^2}\, dw$$

$$= \lim_{b\to\infty} 5\arctan w \Big|_{10^{1/5}}^{b^{1/5}}$$

$$= \lim_{b\to\infty} \left(5\arctan(b^{1/5}) - 5\arctan(10^{1/5})\right)$$

$$\frac{5\pi}{2} - 5\arctan(10^{1/5})$$

8. [12 points] Use the following graph and table to calculate the integrals below.

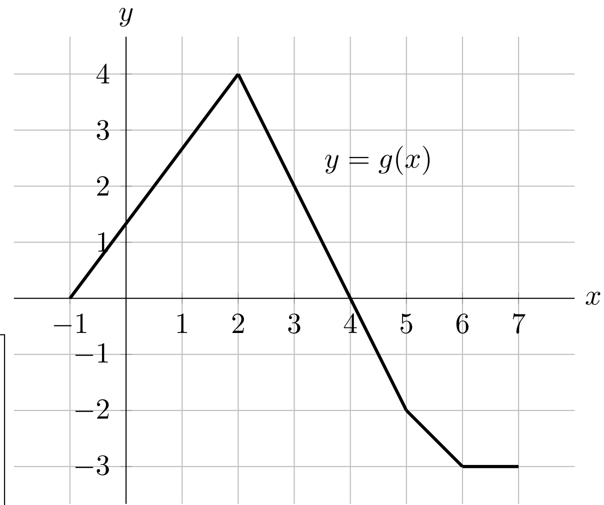
The table below gives several values of a differentiable function f and its derivative f'. Assume that both $f(x)$ and $f'(x)$ are positive and continuous.

x	-2	-1	0	1	3
$f(x)$	0.5	3	4	10	30
$f'(x)$	2	0.5	5	2	22

You are not required to show your work on this problem. However, limited partial credit may be awarded based on work shown.

For each of parts **a.-c.** below, find the exact value of the given quantity. If there is not enough information provided to find the exact value, write "NOT ENOUGH INFO."
All your answers must be in **exact** form.

Let g be the piecewise linear function with graph shown below.



a. [4 points] Find $\int_3^4 tg'(t)\ dt$.

Solution: Use integration by parts, with $u = t$ and $v' = g'(t)$, so $u' = 1$ and $v = g(t)$:

$$\int_3^4 tg'(t)\ dt = tg(t)|_3^4 - \int_3^4 g(t)\ dt$$
$$= 4g(4) - 3g(3) - 1$$
$$= 0 - 6 - 1$$
$$= -7.$$

Alternatively, noting that $g'(t) = -2$ on the interval $[3, 4]$, we have

$$\int_3^4 tg'(t)\ dt = -2\int_3^4 t\ dt = -t^2\Big|_3^4 = -16 + 9 = -7.$$

Answer: -7

b. [4 points] Find $\int_{-1}^1 \frac{2f'(2x+1)}{f(2x+1)}\ dx$.

Solution: Using the substitution $w = f(2x+1)$, so $dw = 2f'(2x+1)$:

$$\int_3^{30} \frac{1}{w}\ dw = \ln|w|\Big|_3^{30}$$
$$= \ln|30| - \ln|3|$$

Answer: $\ln|30| - \ln|3| = \ln|10|$

c. [4 points] Find $\displaystyle\int_1^3 \frac{f'(x)(7f(x)+11)}{(f(x)+1)(2f(x)+4)}\,dx.$

Solution: Starting with the substitution $w = f(x)$, so that $dw = f'(x)\,dx$, the integral becomes

$$\int_{10}^{30} \frac{7w+11}{(w+1)(2w+4)}\,dw$$

We now perform a partial fraction decomposition:

$$\frac{7w+11}{(w+1)(2w+4)} = \frac{A}{w+1} + \frac{B}{2w+4}$$

$$7w+11 = A(2w+4) + B(w+1)$$

Letting $w = -1$ and $w = -2$, we find that

$$4 = 2A \text{ (so } A = 2) \text{ and } -3 = -B \text{ (so } B = 3).$$

Consequently,

$$\int_{10}^{30} \frac{7w+11}{(w+1)(2w+4)}\,dw = \int_{10}^{30} \frac{2}{w+1} + \frac{3}{2w+4}\,dw$$

$$= \left(2\ln|w+1| + \frac{3}{2}\ln|2w+4|\right)\Big|_{10}^{30}$$

$$= 2\ln|31| + \frac{3}{2}\ln|64| - 2\ln|11| - \frac{3}{2}\ln|24|$$

Answer: $2\ln(31) + \frac{3}{2}\ln(64) - 2\ln(11) - \frac{3}{2}\ln(24) = 2\ln(31/11) + \frac{3}{2}\ln(8/3)$

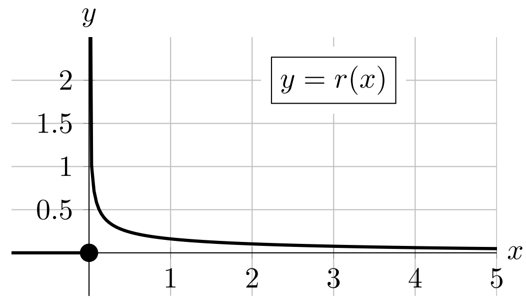
9. [10 points]

It has been suggested that the probability density function given by

$$r(x) = \begin{cases} 0 & \text{if} \quad x \leq 0 \\ \dfrac{e^{-0.1x}}{\sqrt{10\pi x}} & \text{if} \quad x > 0 \end{cases}$$

models the size of rainfalls. That is, on a given rainy day, this pdf models the amount x (measured in millimeters) of rain that falls.

A graph of $y = r(x)$ is shown below.



Note that even though $r(x)$ has a vertical asymptote as $x \to 0^+$, it is still a valid pdf.

a. [1 point] Use the formula above and the fact that $r(x)$ is a pdf to find the value of $\int_0^\infty r(x)\, dx$. (You do not need to show any work.)

Answer: $\int_0^\infty r(x)\, dx =$ 1

b. [4 points] Write out all the terms of a MID(4) approximation to the integral $\int_3^5 r(x)\, dx$. Do not evaluate the sum, but the letters r and x should not appear in your answer.

Solution: With 4 subdivisions, we have $\Delta x = \frac{5-3}{4} = 0.5$. Our four midpoints are at $x = 3.25, x = 3.75, x = 4.25$, and $x = 4.75$. Hence our sum is

$$\frac{e^{(-0.1)(3.25)}}{\sqrt{(10\pi)(3.25)}}0.5 + \frac{e^{(-0.1)(3.75)}}{\sqrt{(10\pi)(3.75)}}0.5 + \frac{e^{(-0.1)(4.25)}}{\sqrt{(10\pi)(4.25)}}0.5 + \frac{e^{(-0.1)(4.75)}}{\sqrt{(10\pi)(4.75)}}0.5$$

or

$$0.5\left(\frac{e^{-0.325}}{\sqrt{32.5\pi}} + \frac{e^{-0.375}}{\sqrt{37.5\pi}} + \frac{e^{-0.425}}{\sqrt{42.5\pi}} + \frac{e^{-0.475}}{\sqrt{47.5\pi}}\right)$$

c. [2 points] Is the answer to part **b.** an overestimate or underestimate of $\int_3^5 r(x)\, dx$? Circle your choice below. You do not need to explain.

Circle one: OVERESTIMATE **[UNDERESTIMATE]** NOT ENOUGH INFORMATION

d. [3 points] Let $q(x)$ be the cumulative distribution function for $r(x)$. Which of the following expressions give the fraction of rainfalls that result in between 2 and 4 millimeters of rain? *Circle* ALL *correct answers.*

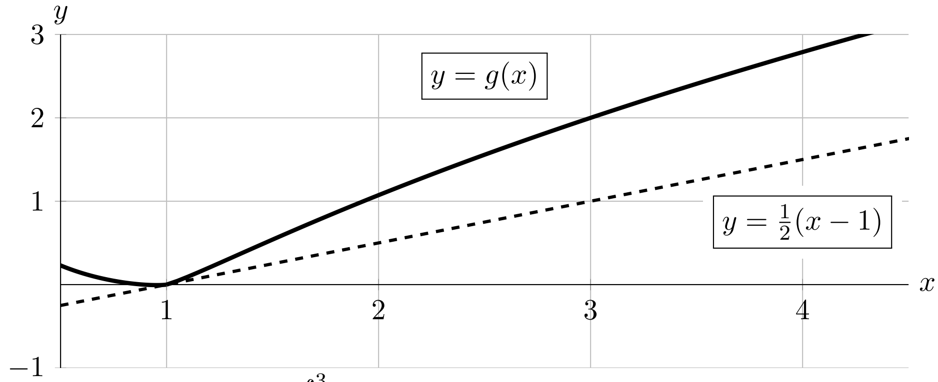
i. $r(4) - r(2)$ ii. $r'(4) - r'(2)$ iii. **[$q(4) - q(2)$]**

iv. $q'(4) - q'(2)$ v. **[$\int_2^4 r(x)\, dx$]** vi. $\int_2^4 r'(x)\, dx$

vii. $\int_2^4 q(x)\, dx$ viii. **[$\int_2^4 q'(x)\, dx$]** ix. NONE OF THESE

10. [7 points] Consider functions f and g that satisfy all of the following:

- $f(x)$ is defined, positive, and continuous for all $x > 1$.
- $\lim_{x\to 1^+} f(x) = \infty$ (so $f(x)$ has a vertical asymptote at $x = 1$).
- $g(x)$ is defined and differentiable for all real numbers x, and $g'(x)$ is continuous.
- $\frac{d}{dx}\left(\frac{g(x)}{\ln x}\right) = f(x)$ for all $x > 1$.
- The tangent line to $g(x)$ at $x = 1$ is given by the equation $y = \frac{1}{2}(x-1)$. Graphs of $g(x)$ (solid) and this tangent line (dashed) are shown below.



Determine whether the integral $\int_1^3 f(x)\,dx$ converges or diverges.

- If the integral converges, circle "Converges", find its exact value, and write the exact value on the answer blank provided.
- If the integral diverges, circle "Diverges" and carefully justify your answer.

Show every step of your work carefully, and make sure that you use correct notation.

Solution: Since $f(x)$ has a vertical asymptote at $x = 1$, we write

$$\begin{aligned}\int_1^3 f(x)\,dx &= \lim_{a\to 1^+}\int_a^3 f(x)\,dx \\ &= \lim_{a\to 1^+} \left.\frac{g(x)}{\ln x}\right|_a^3 \\ &= \lim_{a\to 1^+}\left(\frac{g(3)}{\ln 3} - \frac{g(a)}{\ln a}\right) \\ &= \frac{2}{\ln 3} - \lim_{a\to 1^+}\frac{g(a)}{\ln a} \\ &= \frac{2}{\ln 3} - \lim_{a\to 1^+}\frac{g'(a)}{1/a} \quad \text{where we applied l'Hopital's Rule} \\ &= \frac{2}{\ln 3} - \frac{1/2}{1}\end{aligned}$$

So this improper integral converges.

Circle one:

$\int_1^3 f(x)\,dx$ **converges to** $\frac{2}{\ln 3} - \frac{1}{2}$ or $\int_1^3 f(x)\,dx$ **diverges**

11. [12 points] For each of the questions below, circle all of the available correct answers. Circle "NONE OF THESE" if none of the available choices are correct. No credit will be awarded for unclear markings. No justification is necessary.

a. [4 points] Suppose $f(x)$ is defined and continuous on $(-\infty, \infty)$. Which of the following MUST be true?

i. If a and b are constants with $a \neq b$, then $F(x) = \int_a^x f(t)\,dt$ and $G(x) = \int_b^x f(t)\,dt$ are different functions.

ii. The function $F(x) = \int_a^x f(t)\,dt$ is an antiderivative of $f(x)$ with the property that $F(a) = 0$.

iii. Every antiderivative of $f(x)$ is equal to $\int_c^x f(t)\,dt$, for some choice of constant c.

iv. The function $J(x) = \int_{-x}^2 f(-t)\,dt$ is an antiderivative of $f(x)$.

v. NONE OF THESE

b. [4 points] Suppose $g(t)$ has a positive second derivative for all values of t. Also suppose LEFT(10), RIGHT(10), TRAP(10), and MID(10) are all estimates of the integral $\int_2^5 g(t)\,dt$. Which of the following are POSSIBLE?

i. $\int_2^5 g(t)\,dt < \text{RIGHT}(10)$

ii. $\int_2^5 g(t)\,dt < \text{TRAP}(10)$

iii. $\int_2^5 g(t)\,dt < \text{MID}(10)$

iv. $\text{LEFT}(10) = \text{TRAP}(10) + 100$ and $\text{RIGHT}(10) = \text{TRAP}(10) + 50$

v. $\text{LEFT}(10) = \text{MID}(10) - 100$ and $\text{RIGHT}(10) = \text{MID}(10) - 50$

vi. $\text{LEFT}(10) = \text{MID}(10) - 100$ and $\text{RIGHT}(10) = \text{MID}(10) + 50$

vii. $\text{LEFT}(10) = \text{MID}(10) + 100$ and $\text{RIGHT}(10) = \text{MID}(10) - 50$

viii. NONE OF THESE

c. [4 points] Which of the following are antiderivatives of $h(x) = e^x \cos x$?

i. $J(x) = \int_1^{e^x} \cos(\ln t)\,dt$

ii. $K(x) = \frac{1}{2}e^x \cos x + \frac{1}{2}e^x \sin x + 4$

iii. $L(x) = \int_0^x e^t \cos t\,dt$

iv. $M(x) = \int_0^{x+2\pi} e^{t-2\pi} \cos t\,dt$

v. NONE OF THESE