Math 116 — First Midterm — February 22, 2021

1. You must write your solutions, including all work, for this exam on blank paper and upload you solutions to Gradscope when the exam is over. You may not write your answers on a printed copy of this exam.
2. Write your UMID on the upper right corner of every page you submit. Do not write your name on your submission.
3. This exam has 9 pages including this cover.
4. There are 9 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
5. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
6. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it.
7. The use of any digital or computational device while working on this exam is not permitted. This means you may NOT use a calculator, tablet, computer or other device to do the problems on the exam.
8. For any graph or table that you use to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how you used the graph or table to find the answer.
9. Include units in your answer where that is appropriate.
10. Problems may ask for answers in *exact form*. Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is not.
11. You may use any physical notes or books. These may be handwritten or typed.
12. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	6	
2	13	
3	15	
4	10	
5	10	

Problem	Points	Score
6	15	
7	6	
8	13	
9	12	
Total	100	

1. [6 points] This question is about exam policies. These are the same questions sent to you before the exam. The answer to each is either Yes or No. For each question, write the letter corresponding to the question and the entire word YES or the entire word NO. You do not need to rewrite the questions on your paper.

a. Are you allowed to use any resources, typed or handwritten, by accessing them on a digital device (e.g. computer, tablet, or phone)?

b. Are you allowed to refer to a physical copy of the textbook during the exam?

c. Are you allowed to use a calculator on the exam?

d. If you notice a mistake in one of your answers while you are scanning, is it okay to correct that mistake?

e. Can the penalty for cheating be a failing grade in the course?

f. Is 0.333333333 an exact answer to the equation $3x = 1$?

2. [13 points] Let $f(x) = \dfrac{1}{2x^2+1}$.

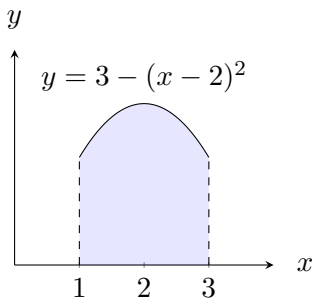
a. [4 points] Approximate the integral $\displaystyle\int_1^5 f(x)\,dx$ using MID(2). Write out each term in your sum. You do not need to simplify the numbers in your sum, but your final answer should not contain the letter "f".

b. [4 points] Approximate the integral $\displaystyle\int_1^5 f(x)\,dx$ using TRAP(2). You do not need to simplify the numbers in your sum, but your final answer should not contain the letter "f".

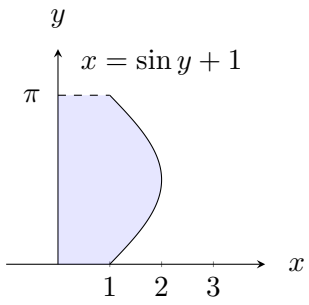
c. [5 points] Compute the exact value for $\displaystyle\int_1^5 f(x)\,dx$. Show all your steps. You do not need to simplify the numbers in your final answer.

3. [15 points] Flora and Nile are collecting fruits in the forest and they have brought several containers of different shapes. **Write an integral** that computes the volume of each of the following containers. **Do not evaluate your integrals.**

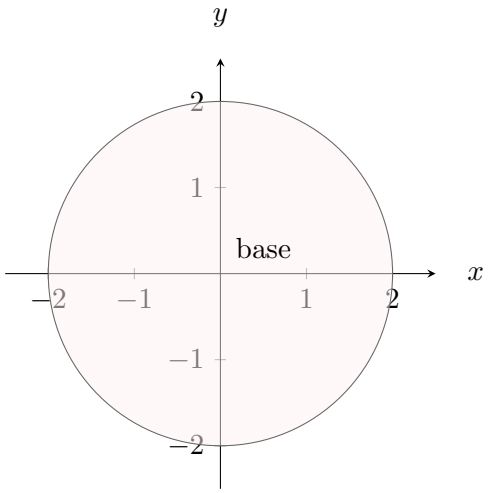
a. [5 points] The first container is in the shape formed by revolving the following region about the y-axis.



b. [5 points] The second container is in the shape formed by revolving the following region about the line $x = 3$.



c. [5 points] The third container has a circular base with equation $x^2 + y^2 = 4$ (of radius 2 centered at the origin), with **square** cross-sections perpendicular to the x-axis.



4. [10 points] After walking in the woods, Flora is making juice with the fruit she picked up at the next hour. The volume of juice in the jar (in gallons) t minutes after she starts making juice is given by the function

$$F(t) = \int_{\sin t}^{2t} \frac{50}{100 - \ln(x+2)}\, dx.$$

a. [3 points] Calculate $F'(t)$.

b. [3 points] What is the volume of juice (in gallons) in the jar when Flora starts making the juice? Briefly explain your answer using the function $F(t)$.

c. [4 points] Nile wants to know the volume of juice in the jar, yet she is confused by the function $F(t)$. She knows she can write $F(t)$ using $F'(t)$ and the initial volume of juice in the jar. Help her by rewriting $F(t)$ in the form

$$F(t) = \int_a^t \_\_\_\_\_\, d\_\_\_\_\_ + \_\_\_\_\_.$$

Write the above integral with the blanks filled in, and also give the value of a.

5. [10 points] Flora pours herself a cup of juice in a cup with the following shape. The cup is filled to the top.

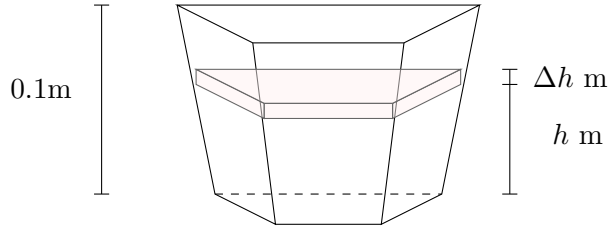


Figure 5.1: Cup

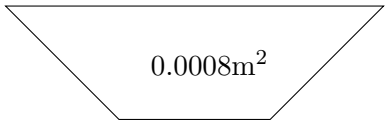


Figure 5.2: Top of cup

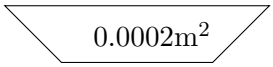


Figure 5.3: Bottom of cup

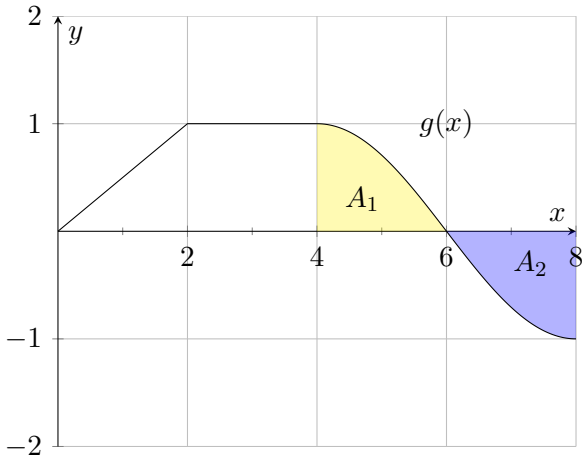
The area of a horizontal cross section of the cup (as shown in Figure 5.1) is **linear** with respect to h, its height above **the bottom of the cup.**
Flora is going to drink the juice with a magical straw. The top of the straw is always 0.05m above **the top of the cup**. Because the straw is magical, it extends automatically and the bottom end of the straw is always at the surface of the juice. The density of the juice is 1100kg/m^3. The gravitational acceleration is $g = 9.8\text{m/s}^2$.

a. [5 points] What is the approximate mass of the slice of juice that is h meters above **the bottom of the cup**, of thickness Δh meters (as shaded in Figure 5.1)? Do not simplify your answer. Include units.

b. [3 points] What is the approximate work needed to lift the same slice of juice (h meters above **the bottom of the cup**, of thickness Δh meters, as shaded in Figure 5.1) to a height of 0.05m above **the top of the cup**? Do not simplify your answer. Include units.

c. [2 points] Write an expression involving integrals for the total work needed to lift all the juice to a height of 0.05m above **the top of the cup**. Do not evaluate any integrals in your expression. Include units.

6. [15 points] Let $g(x)$ be an **odd** function, with part of the graph given as below.



Both shaded regions A_1 and A_2 have area 1.2.
Let $G(x)$ be an antiderivative of $g(x)$ with $G(2) = 2$.

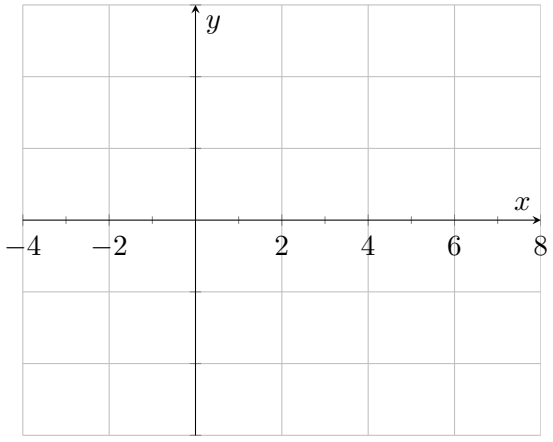
a. [6 points] Copy the following table onto your paper and fill in with the exact values of $G(x)$. You do not need to show your work for this part, but you may receive credit for correct work shown.

x	-4	-2	0	2	4	6	8
$G(x)$				2			

b. [9 points] Sketch a graph of $G(x)$ from $x = -4$ to $x = 8$ on **hand-drawn axes**, similar to those given below. Pay attention to

- if $G(x)$ is increasing / decreasing;
- if $G(x)$ is concave up / concave down / linear;
- all critical points and points of inflection.

Label the (x, y)-coordinates of all the critical points of $G(x)$. If you are worried that the concavity of your drawing is unclear, also label if each portion of your graph is concave up, concave down, or linear.



7. [6 points] Split the function $\dfrac{5x^2 - 7x}{(x-1)^2(x+1)}$ into partial fractions with two or more terms. Do not integrate these terms. Be sure to show all work to obtain your partial fractions.

8. [13 points] Let $f(x)$ be a twice differentiable function with

- $f(0) = 1$.
- $f(\ln 2) = \dfrac{5}{4}$.
- $f'(0) = e$.
- $f'(\ln 2) = 2$.

a. [3 points] Compute the average value of $f'(x)$ on $[0, \ln 2]$.

b. [5 points] Compute the exact value of $\displaystyle\int_0^{\ln 2} x f''(x)\, dx$.

c. [5 points] Compute the exact value of $\displaystyle\int_0^{\ln 2} \frac{f'(x)}{\sqrt{9 - (f(x))^2}}\, dx$.

9. [12 points] For each of the questions below, write out on your paper **all** the answers which are **always** true. For each answer you write, **give an explanation and/or a computation** that shows the statement is always true.

a. [6 points] Let $f(x)$ be a function defined for $0 \le x \le 1$ with $f(x) > 0$ and $f''(x) < 0$. Consider the Riemann sums for the integral $\int_0^1 f(x)\,dx$. Which of the following **must** be true?

$\text{LEFT}(4) \ge \text{RIGHT}(4)$

$\text{MID}(3) \ge \text{TRAP}(2)$

MID(3) is closer to the actual integral than MID(2) is

$\text{LEFT}(3) \le \text{TRAP}(3)$

b. [6 points] Let $g(x)$ be a differentiable function such that

- $0 < g(x) < 1$;
- $g'(x) \neq 0$.

Which of the following **must** be equal to $\dfrac{1}{\sin(g(x))}$?

$\displaystyle\int_0^x \frac{g'(t)}{\sqrt{1-(g(t))^2}}\,dt$

$\displaystyle\frac{d}{dx}\left(-\ln\left(\frac{1}{\sin(g(x))}+\frac{\cos(g(x))}{\sin(g(x))}\right)\right)$

$\displaystyle\frac{1}{g'(x)}\frac{d}{dx}\left(\int_1^{g(x)}\frac{1}{\sin t}\,dt\right)$

$\displaystyle\int_0^x \frac{-g'(t)}{\sin(g(t))\tan(g(t))}\,dt + \frac{1}{\sin(g(0))}$