Math 116 — First Midterm — February 7, 2022

Write your 8-digit UMID number very clearly in the box to the right.

Your Initials Only: _______ Instructor Name: ____________________________ Section #: ______

1. This exam has 9 pages including this cover.
2. There are 8 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it.
5. You are allowed notes written on two sides of a $3'' \times 5''$ note card.
6. For any graph or table that you use to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how you used the graph or table to find the answer.
7. Include units in your answer where that is appropriate.
8. Problems may ask for answers in *exact form.* Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is <u>not</u>.
9. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	16	
2	10	
3	12	
4	11	
5	13	
6	13	

Problem	Points	Score
7	13	
8	12	
Total	100	

1. [16 points] Use the table to compute the following integrals. Write your answer using exact form on the blank provided. If there is not enough information available to answer the question, write N.I. You need to evaluate all integrals, but you do not need to simplify your final answer.

x	0	1	2	3	π	4	8	9
$f(x)$	0	2	5	9	-3	π	6	4
$f'(x)$	-7	-3	4	7	2	1	0	-5
$g(x)$	5	3	-1	2	6	2	-3	e

a. [4 points] $\displaystyle\int_0^4 x^{\frac{1}{2}} g'(\sqrt{x^3})dx$

Answer: ________________

b. [4 points] $\displaystyle\int_4^8 x f''(x)dx$

Answer: ________________

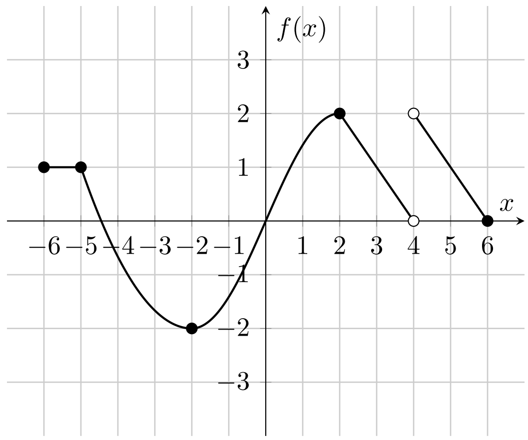
c. [4 points] $\displaystyle\int_0^\pi \cos(t) f(\sin(t))dt$

Answer: ________________

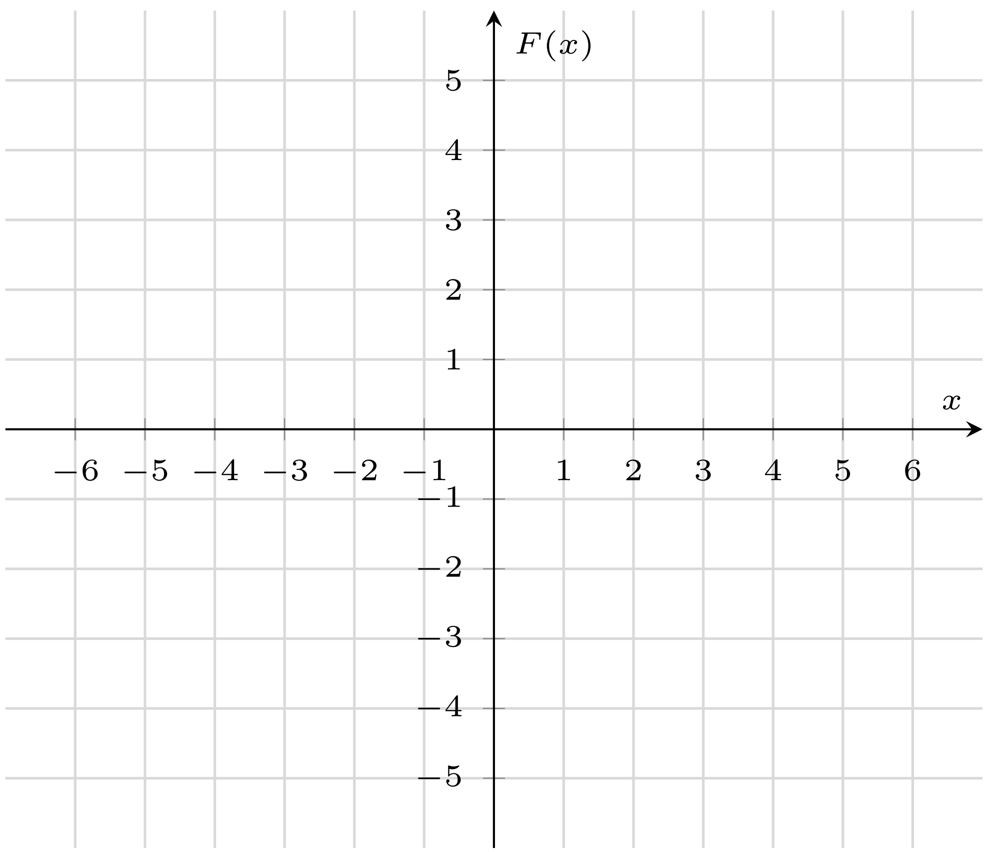
d. [4 points] $\displaystyle\int_4^9 f''(\sqrt{x})dx$

Answer: ________________

2. [10 points]



Sketch the antiderivative $F(x)$ to the function $f(x)$ graphed above, such that $F(2) = -1$. The function $f(x)$ is **odd** on $[-2, 2]$. Make sure to clearly label the input and output values at $x = -2, 2, 5$, and 6. Be sure to make it clear where the graph is concave up, concave down, or linear, and where it is increasing or decreasing.



3. [12 points]

a. [6 points] Split the following function into partial fractions. Do not integrate the result. Be sure to show all your work.

$$\frac{10x+2}{(x-1)^2(x+2)}$$

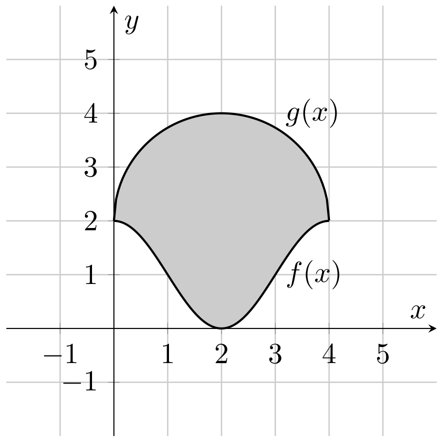
b. [6 points] Given the partial fraction decomposition

$$\frac{-x-10}{(x-3)(x^2+4)} = \frac{-1}{(x-3)} + \frac{x+2}{x^2+4},$$

evaluate the following indefinite integral, show all your steps:

$$\int \frac{-x-10}{(x-3)(x^2+4)} dx$$

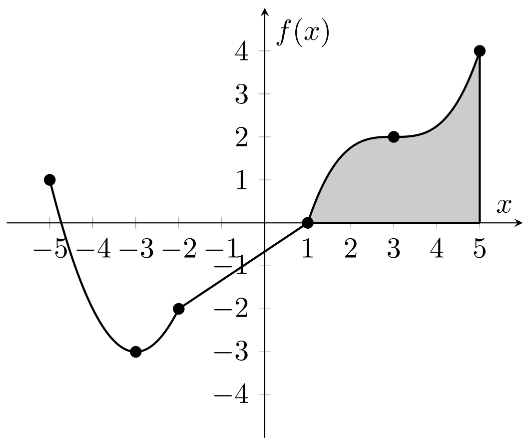
4. [11 points] Consider the shaded region bounded by $f(x) = 2\cos^2\left(\frac{\pi x}{2}\right)$ and $g(x) = \sqrt{4-(x-2)^2} + 2$ shown below.



a. [6 points] Write, but do not compute, an integral for the solid formed by rotating the region around the line $x = 5$.

b. [5 points] Write, but do not compute, an expression involving one or more integrals for the perimeter of the region above. *Hint: The upper curve is a semicircle.*

5. [13 points] The function below has a local minimum at $x = -3$, is linear on $[-2, 1]$, and has an inflection point at $x = 3$.



For parts **a.** and **b.**, use the graph of $f(x)$ to determine if the listed quantities are over- or under-estimates for the relevant integral, and write the word OVERESTIMATE or UNDERESTIMATE as appropriate. If there is not enough information, write NI.

a. [4 points] $\displaystyle\int_{-3}^{3} f(x)dx$

LEFT(4) ________________ RIGHT(4) ________________

MID(4) ________________ TRAP(4) ________________

b. [4 points] $\displaystyle\int_{-5}^{1} f(x)dx$

LEFT(12) ________________ RIGHT(12) ________________

MID(12) ________________ TRAP(12) ________________

c. [5 points] The function on $[1, 5]$ is defined by $\frac{1}{4}(x-3)^3 + 2$. Write, but do not solve, an integral giving the volume of the shaded region rotated around $y = -2$.

6. [13 points]

a. [4 points] Which, if any, of the following are antiderivatives of the function e^{x^2}? Circle ALL that apply, or 'NONE' as appropriate.

$$\int_1^{x^2} \frac{e^t}{2t} dt \qquad \int_{\sqrt{2}}^{x} e^{t^2} dt \qquad \frac{e^{x^2}}{2x}$$

$$\int_0^{x^2} e^t dt \qquad \text{NONE}$$

b. [4 points] Back in his younger days, Brad hiked to the top of Mount Olympus. Let the continuous function $T(m)$ denote the rate of change of temperature, in degrees Celsius per meter, after Brad has hiked m meters. Suppose the following mathematical statements hold:

- $\int_0^{2500} T(m)dm = -25.$
- $-0.01 = \frac{1}{1000} \int_0^{1000} T(m)dm.$

Which of the following statements MUST be true? Circle ALL that apply, or 'NONE OF THE ABOVE' as appropriate.

i) $T(m)$ is negative for all values of m in its domain.

ii) The average rate of change of temperature per meter hiked was the same during the first 1000 meters Brad hiked as it was in the next 1500 meters he hiked.

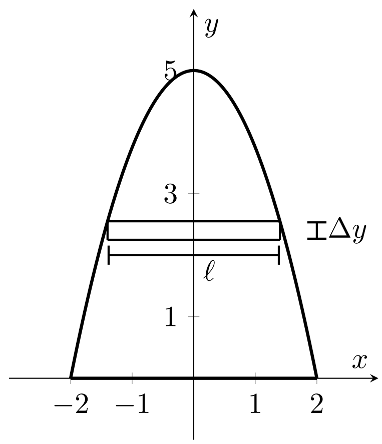
iii) During the first 1000 minutes Brad was hiking, the temperature decreased by an average of 0.1 degrees Celsius per minute.

iv) The temperature decreased by 10 degrees Celsius during the first 1000 meters of Brad's hike.

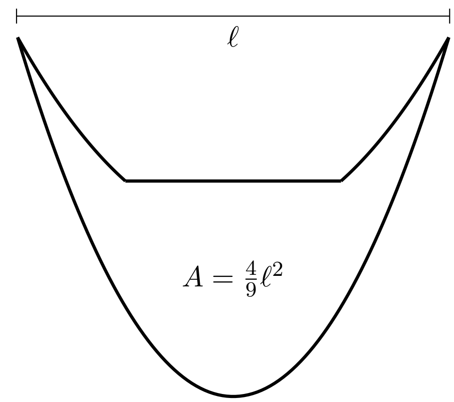
v) NONE OF THE ABOVE

c. [5 points] What is the mass of a solid cube with side length ℓ centimeters if its density x centimeters above its base is $x + 1$ grams per cubic centimeter for $0 \le x \le \ell$. Show all your work including evaluating all integrals and give your answer in terms of ℓ.

7. [13 points] Brad and Shawna are shipwrecked on an island and are building a new ship out of various materials. The ship has a base given by the region enclosed in the figure on the left, with cross-sections perpendicular to the y-axis given by the figure on the right. The base is the region bounded by $y = \frac{-5}{4}(x^2 - 4)$ and $y = 0$. The cross-sections have area given by $\frac{4}{9}\ell^2$ where ℓ is the length of the slice of the base directly below the cross-section. A sample slice of the base of thickness Δy is shown in graph on the left, and all distances are given in meters.



Base of Ship



Cross-section of Ship

a. [3 points] Write an expression for the length, ℓ, of a the slice y meters from the x-axis. Give units.

b. [3 points] Write an expression for the volume of materials needed to construct a cross-sectional slice of the ship y meters from the x-axis with thickness Δy meters. The letter ℓ should not appear in your final answer. Give units.

c. [3 points] The density of the materials used to make the ship varies. The materials used in the cross section y meters from the x-axis is given by $\delta(y) = (2y + 5)\ kg/m^3$. What is the mass of a cross sectional slice y meters from the x-axis with thickness Δy meters? Give units.

d. [4 points] Write an integral that gives the total mass of the new boat in kg. Do not evaluate your integral.

8. [12 points] After constructing their boat, Brad and Shawna departed the island, hoping to return home. However, their victory at Troy angered Poseidon, the god of the sea, who created large waves to further complicate their journey. The waves were so high Brad and Shawna could not see land, making navigation difficult. Hermes, the messenger of the gods, was sympathetic. He stole the formula Poseidon used to create the waves and gave it to Brad and Shawna. The formula is given by:

$$H(t) = 12\int_0^{\sin^2(t)} e^{x^2}\,dx.$$

The function $H(t)$ gives the wave height in meters, t minutes after Hermes stole the formula.

a. [7 points] Brad and Shawna can only see land at the moment their ship is at the top of a wave. If they know exactly when that time is coming, they can be prepared to correct their course toward land. If Hermes brought them the formula 4 minutes after he stole it, when is the first time they can see land? (Brad and Shawna did not know when to look for land before obtaining the formula.)

b. [5 points] The same poet who recorded the tale of the Trojan war would like to record parts of Brad and Shawna's odyssey. Seeking a more appealing version of the expression above, he asks you for a different formula. Write a function equivalent to $H(t)$ with only t in the upper bound of the integral.