1. (7 points) The *sine-integral* function $Si(x)$ is defined by

$$Si(x) = \int_0^x \frac{\sin t}{t} \, dt.$$

What is the derivative of $Si(x^3)$?

*Answer:* We use the 2nd Fundamental Theorem and and the chain rule to arrive at our answer. According to that theorem, the derivative of $Si(x)$ is $Si'(x) = \sin(x)/x$. Therefore, by the chain rule,

$$\frac{d}{dx}Si(x^3) = Si'(x^3) \frac{d}{dx}x^3 = \frac{\sin(x^3)}{x^3} \times 3x^2 = \frac{3\sin(x^3)}{x}.$$

2. (10 points) Let $g(x)$ be a continuously differentiable functions of $x$ that satisfies $g(1) = 2$, $g(5) = 6$, and $\int_1^5 g(x) \, dx = -2$. Compute, showing all your work.

*Answers:*

(a) We use integration by parts to compute this integral:

<table>
<thead>
<tr>
<th>$u = x$</th>
<th>$dv = g'(x) , dx$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$du = dx$</td>
<td>$v = g(x)$</td>
</tr>
</tbody>
</table>

$$\int_1^5 xg'(x) \, dx = xg(x) \bigg|_1^5 - \int_1^5 g(x) \, dx$$

$$= (5g(5) - 1g(1)) - (-2)$$

$$= 30.$$

(b) We use a $u$-substitution for this integral. Let $u = 4x - 7$, so $du = 4 \, dx$.

$$\begin{array}{c|c|c}
\hline
x & u & \\
\hline
2 & 1 & \\
3 & 5 & \\
\hline
\end{array}$$

$$\int_2^3 g(4x - 7) \, dx = \frac{1}{4} \int_1^5 g(u) \, du$$

$$= -\frac{2}{4}$$

$$= -\frac{1}{2}$$

3. (6 points) Let $r(t)$ represent the rate that the height of a child changes per year (in inches per year), where $t = 0$ corresponds to the birth date of the child. Explain the meaning of the quantity $\int_4^8 r(t) \, dt$. (Remember to use units.)

*Answer:*

The quantity $\int_4^8 r(t) \, dt$ represents the number of inches a child grows between 4 years of age and 8 years of age.