1. (7 points) The *sine-integral* function $Si(x)$ is defined by

$$Si(x) = \int_0^x \frac{\sin t}{t}\, dt.$$

What is the derivative of $Si(x^3)$?
Answer: We use the 2nd Fundamental Theorem and and the chain rule to arrive at our answer. According to that theorem, the derivative of $Si(x)$ is $Si'(x) = \sin(x)/x$. Therefore, by the chain rule,

$$\begin{aligned} \frac{d}{dx} Si\left(x^3\right) = Si'(x^3)\frac{d}{dx}x^3 &= \frac{\sin(x^3)}{x^3} \times 3x^2 \\ &= \frac{3\sin(x^3)}{x} \end{aligned}$$

2. (10 points) Let $g(x)$ be a continuously differentiable functions of x that satisfies $g(1) = 2$, $g(5) = 6$, and $\int_1^5 g(x)\, dx = -2$. Compute, showing all your work,
Answers:
(a) We use integration by parts to compute this integral:

$u = x$	$dv = g'(x)dx$
$du = dx$	$v = g(x)$

$$\begin{aligned} \int_1^5 xg'(x)dx &= xg(x) \mid_1^5 - \int_1^5 g(x)dx \\ &= (5g(5) - 1g(1)) - (-2) \\ &= 30. \end{aligned}$$

(b) We use a u-substition for this integral. Let $u = 4x - 7$, so $du = 4dx$.

x	u
2	1
3	5

$$\begin{aligned} \int_2^3 g(4x-7)dx &= \frac{1}{4}\int_1^5 g(u)du \\ &= -\frac{2}{4} \\ &= -\frac{1}{2} \end{aligned}$$

3. (6 points) Let $r(t)$ represent the rate that the height of a child changes per year (in inches per year), where $t = 0$ corresponds to the birth date of the child. Explain the meaning of the quantity $\int_4^8 r(t)\, dt$. (Remember to use units.)
Answer:
The quantity $\int_4^8 r(t)dt$ represents the number of inches a child grows between 4 years of age and 8 years of age.