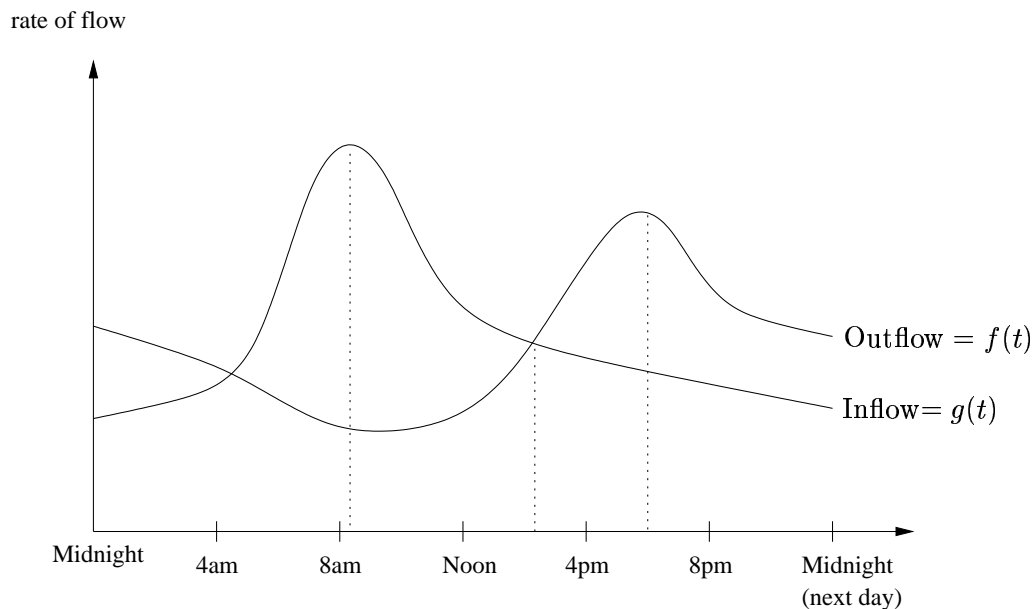


10. (15 points) The graphs shown represent the flow of traffic (in number of motor vehicles per minute) in and out of Ann Arbor on a typical weekday.



(a) During the course of the day, at what time is the largest number of cars in Ann Arbor? Give an explanation of how you arrived at this answer.

Answer:

The number of cars at time  $T$  is equal to the initial number of cars plus  $\int_0^T (g(t) - f(t)) dt$ . In order to find out when the largest number of cars are in Ann Arbor, we want to find when this integral is the largest. This happens around 2:30 pm. The number of cars decreases slightly between midnight and 4:15 am, but the amount of increase between 4:15 am and 2:30 more than makes up for this decrease. After 2:30 the integrand is negative, so the number of cars begins to decrease.

(b) At what time is the number of cars in Ann Arbor increasing the most rapidly? Decreasing the most rapidly? Again, please give an explanation of how you arrived at this answer.

Answer:

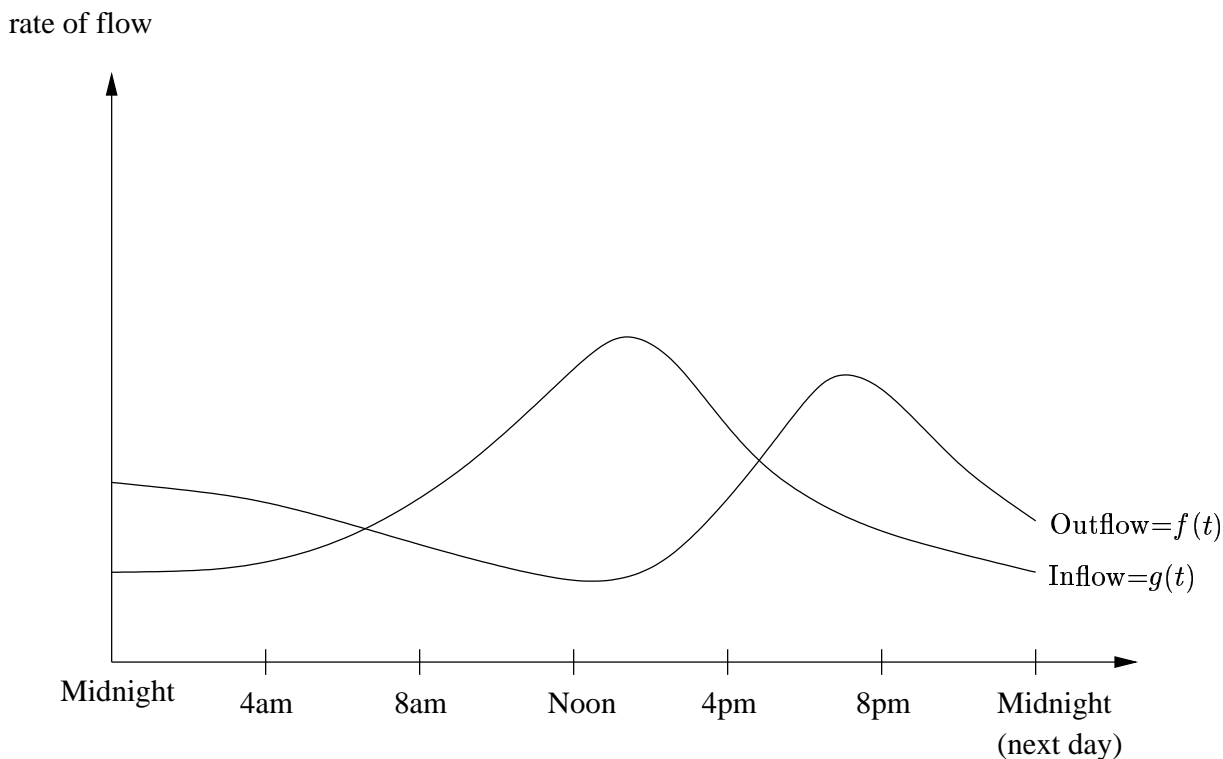
The number of cars is increasing most rapidly when the rate in - rate out is the largest. This happens at about 8:15 am.

The number of cars is decreasing most rapidly when the rate out - rate in is the largest. This happens at about 6:00 pm.

(c) Sketch possible graphs of the inflow of traffic and the outflow of traffic in for Ann Arbor on a football Saturday if we assume kickoff is at 3pm. Explain how you arrived at the graph drawn.

Answer:

There are many possible answers here, what we want is something reasonable with a good explanation.



Some people will want to leave town early in the morning to get out of the craziness of a football Saturday in Ann Arbor. Around noon people will start coming into town to tailgate and beat the rush. The maximum rate of cars entering will be around 1:30 pm, giving people some time to park and get to their seats before kickoff. The outflow is very low once people start arriving because the people of Ann Arbor know better than to venture from their homes once football traffic has started. The point where the outflow and inflow cross is the time when the maximum number of cars are in Ann Arbor, as we saw in part (a). This occurs at a little after 4 pm. This allows for people who arrived late to the game as well. One could put this at 3pm or around that time and be fine as well. The outflow will start becoming large at about 7 pm as people leave the game early or as the game ends. The maximum outflow will be smaller than the maximum inflow as people will hang around after the game to go to the bars and visit friends.