5. (10 points) Circle the correct answer(s) to each of the following questions or circle True or False, as appropriate. (No explanations necessary.) In all the questions, $f$ is a continuous function defined on an interval $a \leq x \leq b$.

(a) If $f$ is a decreasing function, then for every $n = 1, 2, 3, \ldots$, the approximation to $\int_a^b f(x) \, dx$ given by LEFT$(n)$ is an

- underestimate.  
- overestimate.  

could be either.

(b) If $f$ is concave down, then for every $n = 1, 2, 3, \ldots$, the approximation to $\int_a^b f(x) \, dx$ given by TRAP$(n)$ is an

- underestimate.  
- overestimate.  

could be either.

(c) If $n$ is very large, then the midpoint rule MID$(n)$ always gives one the exact value of $\int_a^b f(x) \, dx$.

- True.  
- False.

(d) The approximation to $\int_a^b f(x) \, dx$ given by the trapezoidal rule TRAP$(n)$ is always more accurate than that given by the left rule, LEFT$(n)$.

- True.  
- False.

(e) Given a graph of $f'(x)$, one can uniquely determine the graph of $f(x)$.

- True.  
- False.

6. (10 pts.) A leak is found in the dam your company just finished constructing. The reservoir behind the dam contains 10 million gallons of water when the leak is first discovered, and it is believed that the water is leaking out at a rate of $r(t) = 0.23e^{\frac{t}{4}}$ millions of gallons per hour $t$ hours after this time. If it takes your crew 5 hours to repair the leak, how much water would be lost? (Be sure to show your work and explain how arrived at your answer.)

Answer:
The amount of water that has leaked out after 5 hours is equal to the integral from $t = 0$ to $t = 5$ of the rate that water leaks from the dam, or

$$\int_0^5 0.23e^{\frac{t}{4}} \, dt.$$ 

This integral is not readily computable by hand, so one uses a calculator to get an approximate numerical answer. Using the numerical integration function on a TI-83, the result, to two decimal places is

$$\int_0^5 0.23e^{\frac{t}{4}} \, dt \approx 2.22 \text{ millions of gallons.}$$

So after 5 hours, only $10 - 2.22 = 7.78$ millions of gallons remain in the reservoir.