7. (10 points) A function $F$ is defined for $-2 \leq x \leq 4$ by the formula
\[
F(x) = \int_0^x e^{-h(t)} \, dt
\]
where $h$ is the function with the graph shown below.

(a) True or False? For $-2 < x < 4$, $F'(x) = e^{-h(x)}$. T \[F\]

(b) On which interval or subintervals of $[-2, 4]$ is $F$ increasing? decreasing?
Answer:
Using the second fundamental theorem we see that
\[
F'(x) = e^{-h(x)}
\]
for all $x \in [-2, 4]$. But we know $e^t > 0$ for all $t$, so this means that the derivative of $F(x)$ is always positive. Thus $F(x)$ is increasing on the entire interval $[-2, 4]$.

(c) On which interval or subintervals of $[-2, 4]$ is $F$ concave up? concave down?
Answer:
To check concavity, we take the second derivative of $F(x)$:
\[
F''(x) = -h'(x) e^{-h(x)}.
\]
From the graph we see $h(x)$ is an increasing function on $[-2, 4]$; so $h'(x) > 0$ on $[-2, 4]$. Thus $-h'(x) < 0$ on $[-2, 4]$. Using again that $e^t > 0$ always, we get that $F''(x) < 0$ on $[-2, 4]$ and so $F(x)$ is concave down on the entire interval.
Alternatively, we can observe that $F'(x) = e^{-h(x)}$ is a decreasing function because $h$ and the exponential function are increasing. Therefore, $F$ is concave down.