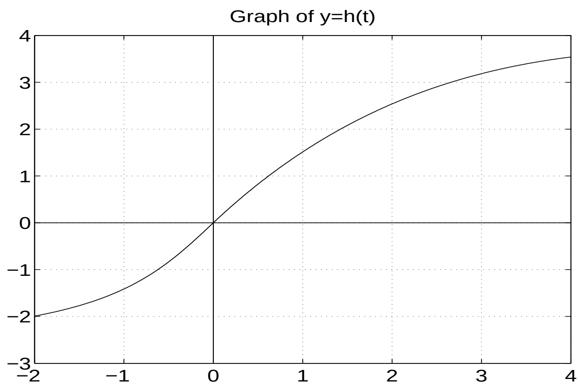
7. (10 points) A function F is defined for $-2 \le x \le 4$ by the formula

$$F(x) = \int_0^x e^{-h(t)}\,dt$$

where h is the function with the graph shown below.



(a) True or False? For $-2 < x < 4$, $\quad F'(x) = e^{-h(t)}$. T $\boxed{\text{F}}$

(b) On which interval or subintervals of $[-2, 4]$ is F increasing? decreasing?

Answer:

Using the second fundamental theorem we see that

$$F'(x) = e^{-h(x)}$$

for all $x \in [-2, 4]$. But we know $e^t > 0$ for all t, so this means that the derivative of $F(x)$ is always positive. Thus $F(x)$ is increasing on the entire interval $[-2, 4]$.

(c) On which interval or subintervals of $[-2, 4]$ is F concave up? concave down?

Answer:

To check concavity, we take the second derivative of $F(x)$:

$$F''(x) = -h'(x)e^{-h(x)}.$$

From the graph we see $h(x)$ is an increasing function on $[-2, 4]$, so $h'(x) > 0$ on $[-2, 4]$. Thus $-h'(x) < 0$ on $[-2, 4]$. Using again that $e^t > 0$ always, we get that $F''(x) < 0$ on $[-2, 4]$ and so $F(x)$ is concave down on the entire interval.

Alternatively, we can observe that $F'(x) = e^{-h(x)}$ is a decreasing function because h and the exponential function are increasing. Therefore, F is concave down.